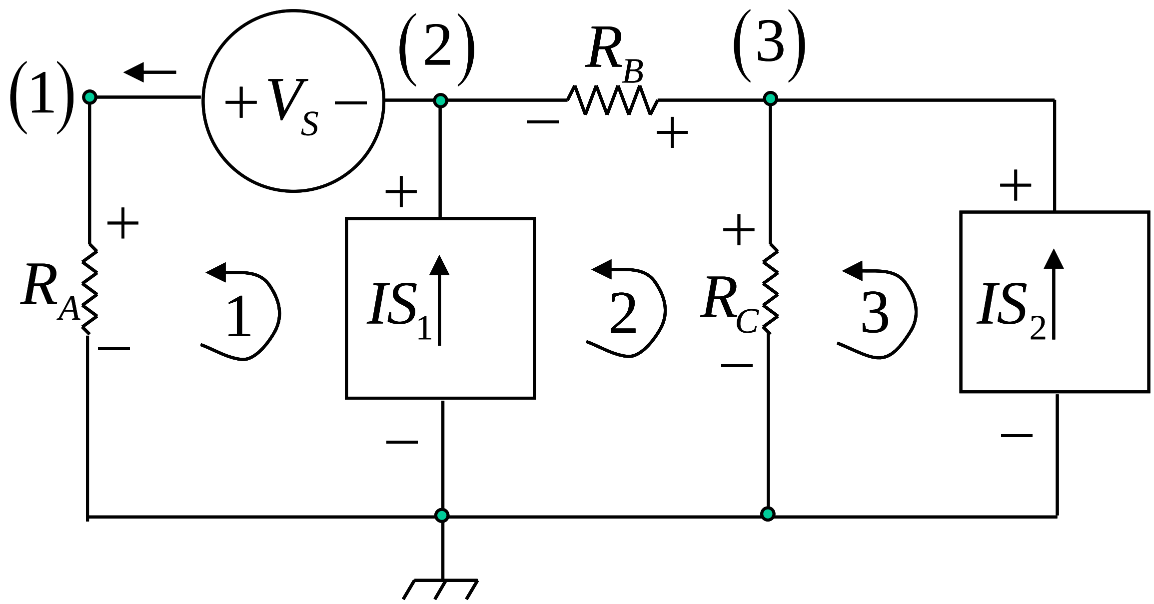
Problem 1 (25%)

Analyze the circuit in the figure using Ohm's law and Kirchoff's voltage and current laws. The given parameters are in the equation below the figure. Give the solution as the voltages at nodes 1, 2, and 3 in the figure. You may use superposition.



$$R_A = 40\ k\Omega \quad R_B = 5\ k\Omega \quad R_C = 5\ k\Omega$$
$$V_S = 10\ V \quad IS_1 = 1\ mA \quad IS_2 = 2\ mA$$

Hint A: Solve the circuit using only one source at a time by setting the other two sources to zero, and then add the three solutions.

Hint B: A voltage source with a voltage of zero is a short circuit – the exact same thing as a wire going thorough it.

Hint C: A current source with a current of zero is an open circuit – the exact same thing as if the terminals to and from it were removed from the circuit.

Solution

This problem was modified from homework problem 5.3-2 page 187 of the text, *Introduction to Electric Circuits*, 7th Edition, by Dorf and Svoboda. The homework problem asked for one current, i_A, while the quiz problem asks that the entire circuit to be solved. Given i_A, the entire circuit is solved with Ohm’s law. The values were changed from those of the homework problem so that the solution is not exactly the same as that of the homework problem.

The text specifies that superposition be used to solve the circuit, but any of several methods will work well. For the quiz, I will accept any method. For example, the circuit configuration is that of a ladder circuit, so successive transformations between Norton and Thévenin circuits from the right will reduce the circuit to a source and resistance in series with R_A, so that i_A, is found directly using Ohm's law.

Note that V_1 and V_2 are a supernode, and indeed unless V_S is nonzero they are the same node. We will use superposition here to find V_2 and V_3 due to each of the three sources, add them, and used these voltages to find the currents.

When V_S is the only source, we have

$$VV_2 = -\frac{R_B + R_C}{R_A + R_B + R_C} \cdot V_S = -\frac{5000 + 5000}{40000 + 5000 + 5000} \cdot 10 = -2\,V$$

$$VV_1 = VV_2 + V_S = 8\,V$$

$$VV_3 = -\frac{R_C}{R_A + R_B + R_C} \cdot V_S = -\frac{5000}{40000 + 5000 + 5000} \cdot 10 = -1\,V$$

When IS_1 is the only nonzero source, we can use the Thévenin equivalent, which is a voltage source of $IS_1 \cdot R_A$ driving a voltage divider of R_A, R_B and R_C, and we have

$$IS1V_1 = IS1V_2 = \frac{R_B + R_C}{R_A + R_B + R_C} \cdot IS_1 \cdot R_A = 8\,V$$

$$IS1V_3 = \frac{R_C}{R_A + R_B + R_C} \cdot IS_1 \cdot R_A = 4\,V$$

When IS_2 is the only nonzero source, we can use the Thevenin equivalent of a voltage source in series with R_C in place of R_C and IS_2, and we again have a simple voltage divider:

$$IS2V_1 = \frac{R_A}{R_A + R_B + R_C} \cdot IS_2 \cdot R_C = 8\,V$$

$$IS2V_3 = \frac{R_A + R_B}{R_A + R_B + R_C} \cdot IS_2 \cdot R_C = 9\,V.$$

The sum of the voltages gives us

$$V_1 = VV_1 + IS1V_1 + IS2V_1 = 24\,V$$

$$V_2 = VV_2 + IS1V_2 + IS2V_2 = 14\,V$$

$$V_3 = VV_3 + IS1V_3 + IS2V_3 = 12\,V$$

The resistor currents are found by Ohm's law.

Problem 2 (25%)

The circuit and parameters below are the same as for Problem 1. Write the Ohm's law and Kirchoff loop and node equations. Then rearrange them as a square matrix of constants and resistances, left-multiplying a vector of unknowns, equal to a vector of knowns. Write down the matrix of the solution in the form

$$M = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{bmatrix}$$

$$M \cdot \begin{bmatrix} Unk_1 \\ Unk_2 \\ Unk_3 \\ Unk_4 \\ Unk_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$

where the b_i are linear combinations of the sources, or zero. The solution required for this problem is the elements of the matrix M, the unknowns Unk_i that you selected, and the source contributions b_i. Numerical values are not required, and **don't invert the matrix.** Label the rows by the Ohm's law, node, or loop equation so that I can compare them with your work in grading this problem.

Hint: Use voltage node notation and leverage supernodes. Note that some loops are trivial in simple circuits like this. You may use Ohm's Law can be used to couple unknown currents into a loop equation.

Solution

The loops and nodes are marked on the figure for Problem 1. Those of you who solved Problem 1 with the matrix method need only present their matrix form organization of the node and loop equations here to answer Problem 2.

Nodes 1 and 2 are a supernode because they are separated by voltage source V_S. The Ohm's law equations are

$$\frac{V_1}{R_A} = \frac{V_2 + V_S}{R_A} = i_A$$

$$\frac{V_3 - V_2}{R_B} = i_B$$

$$\frac{V_2}{R_C} = i_C$$

The voltage equation for Loop 1 is

$$\left(0-V_2\right)-V_S+V_1=0$$

or,

$$-VIS_1-V_S+V_1=0.$$

The voltage loop equation for Loop 2 is

$$\left(0-V_3\right)+\left(V_3-V_2\right)+VIS_1=0$$

or, with Ohm's law for $R_{B,}$

$$-V_3+i_B\cdot R_B+VIS_1=0.$$

The voltage loop equation for Loop 3 is

$$-VIS_2+V_3=0$$

or, with Ohm's law for R_C,

$$-VIS_2+i_C\cdot R_C=0.$$

The sum of the currents into Nodes 1 and 2 is

$$-\frac{V_S+VIS_1}{R_A}+\frac{V_3-V_2}{R_B}+IS_1=0.$$

The sum of the currents into Node 3 is

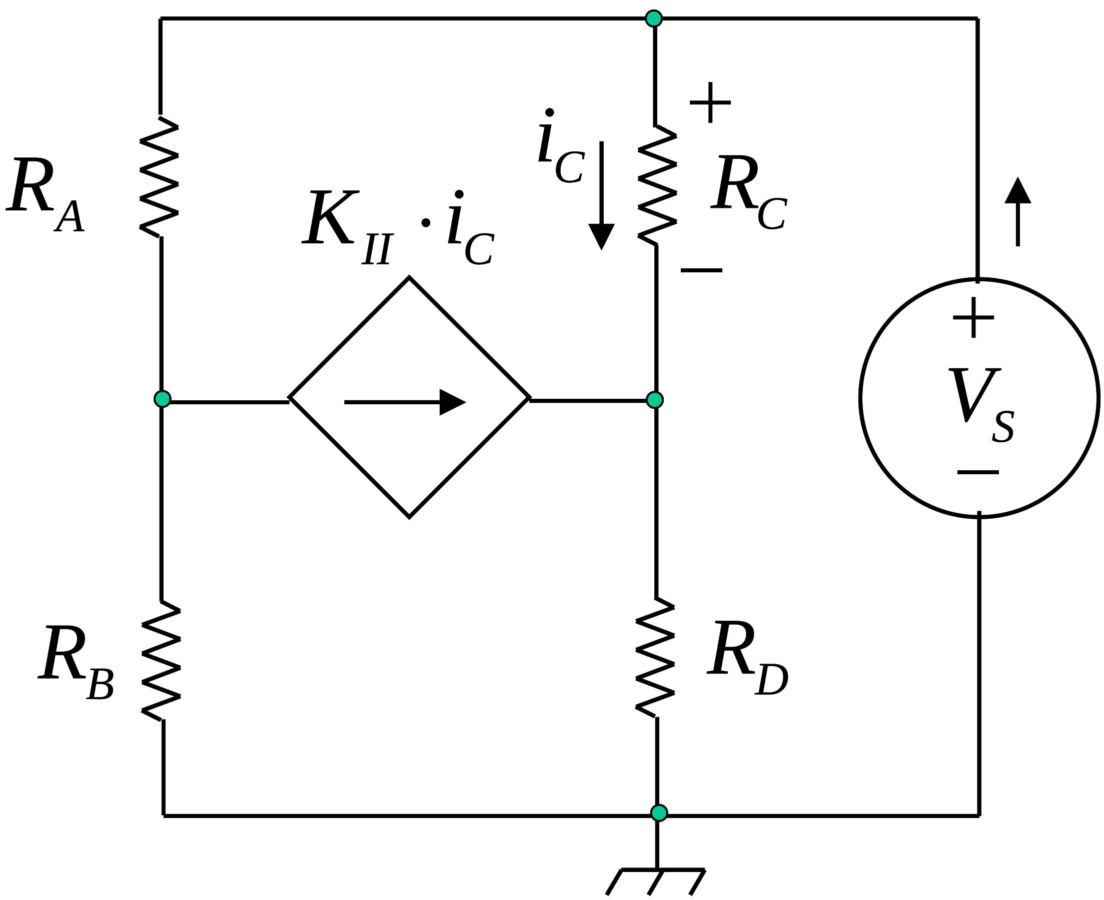
$$-\frac{V_3-V_2}{R_B}-\frac{V_3}{R_C}+IS_2=0.$$

There are many ways to produce a simple reorganization that defines a solution of variables that solves the circuit. One such uses Ohm's law for R_A and R_B, and the current node equations:

$$\begin{array}{l} \\ \text{Ohm's law for } R_A \\ \text{Ohm's law for } R_B \\ \text{Nodes 1 and 2} \\ \text{Node 3} \end{array} \quad \begin{array}{c} \begin{array}{cccc} V_2 & V_3 & i_A & i_B \end{array} \\ \begin{bmatrix} \frac{1}{R_A} & 0 & 1 & 0 \\ 1 & -1 & 0 & R_B \\ -\frac{1}{R_A}-\frac{1}{R_B} & \frac{1}{R_B} & 0 & 0 \\ \frac{1}{R_B} & -\frac{1}{R_B}-\frac{1}{R_C} & 0 & 0 \end{bmatrix} \end{array} \cdot \begin{bmatrix} V_2 \\ V_3 \\ i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -\frac{V_S}{R_A} \\ 0 \\ \frac{V_S}{R_A}-IS_1 \\ -IS_2 \end{bmatrix}$$

Problem 3 (25%)

Solve the circuit in the figure for the parameters given below the figure.



$$V_S = 12\,V \qquad K_{II} = 3 \qquad R_A = 10\,\Omega$$
$$R_B = 20\,\Omega \quad R_C = 40\,\Omega \quad R_D = 15\,\Omega$$

Hint A: Look for parts of the circuit that operate without dependence on other parts.

Hint B: The solution of the circuit is completely characterized by the voltages on either side of the controlled current source.

Solution

This problem was modified from the text's homework problem.4.7-8 page 150. The values were changed and the problem stops at solving the circuit and does not ask for the power provided by the dependent source, as does the homework problem.

From the first hint, we look for parts of the circuit that operate independently of the rest. We see that the controlled current source, R_C and R_D operate independently of R_A and

R_B. From the second hint we see that the circuit is completely characterized by the voltages on either side of the controlled current source, and we see that, given these voltages, the resistor currents are found by Ohm's law and the circuit is completely solved. So, we call the node between R_A and R_B Node 1 and its voltage V_1, and the node between R_C and R_D Node 2 and its voltage V_2. The sum of the currents into Node 2 is

$$\frac{V_S - V_2}{R_C} + K_{II} \cdot \frac{V_S - V_2}{R_C} - \frac{V_2}{R_D} = 0\,.$$

We divide both sides by V_2 and collect terms on the left hand side to get a simpler form

$$\frac{1+K_{II}}{R_C} \cdot \left(\frac{V_S}{V_2} - 1 \right) = \frac{1}{R_D}$$

or,

$$\frac{V_S}{V_2} = 1 + \frac{R_C}{(1+K_{II}) \cdot R_D}$$

from which we see that

$$V_2 = \frac{V_S}{1 + \frac{R_C}{(1+K_{II}) \cdot R_D}}\,.$$

The controlled current source is

$$K_{II} \cdot i_C = K_{II} \cdot \frac{V_S - V_2}{R_C}$$

$$= \frac{K_{II} \cdot V_S}{(1+K_{II}) \cdot R_D + R_C}\,.$$

Given the current source, we can write the currents into Node 1 as

$$\frac{V_S - V_1}{R_A} - K_{II} \cdot i_C - \frac{V_1}{R_B} = 0\,.$$

We get V_1 from superposition of V_S and the controlled current source as

$$V_1 = \frac{R_B}{R_A + R_B} \cdot V_S - \frac{R_A \cdot R_B}{R_A + R_B} \cdot K_{II} \cdot i_C\,.$$

We use the given circuit parameters to find the values,

$$V_2 = 7.2\,V$$

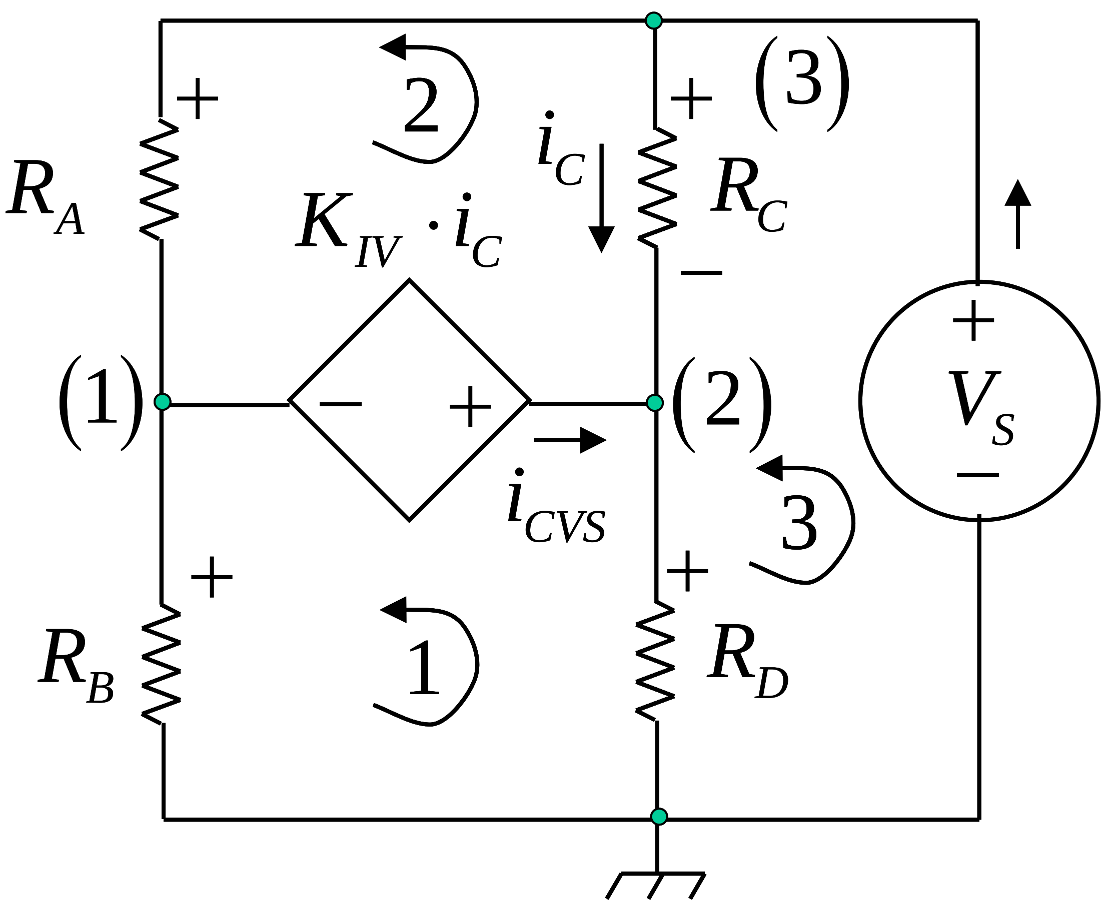
$$i_C = 0.12\ A$$

$$V_1 = 5.6\,V$$

and the rest of the circuit is found using Ohm's law.

Working the Wrong Problem

A significant number of you took the controlled current source as a controlled voltage source and worked the problem in the figure below. If this is done, there are no parts of the circuit that are not dependent on other parts, so Hint 1 does not apply. No one asked whether this was a controlled voltage or current source. I wasn't concerned because the use of "+" and "-" signs inside the circle, square, or diamond indicate a voltage source while an arrow indicates a current source is the main point of Slide 3 of the review session given the morning before the quiz, and prepared as a handout that you were told to bring to the quiz. However, since a significant number of you did work the wrong problem, I need to have a detailed solution available in order to give you partial credit. Also, this particular problem is an example of a simple circuit that does not yield easily to intuitive methods and is best solved using the Kirchhoff's laws approach. The problem, as mis-stated, is similar to problem 4.4-8 page 145, which is a long problem.



The control gain K_{II} is rewritten K_{IV} to denote the misconstruing the controlled source as a voltage source and to flag the gain as having the units of resistance for consistency checks in the equations. We honor the passive sign convention (slide 1 in the quiz review and handout) and the source sign convention (slide 2), use node voltages as unknowns (slide 5), write Ohm's law first (slide 5), write small loops all in the same direction (slide

4), and add voltage drops around each loop (slides 6 and 7). Loop equations are written as sums of voltage drops, followed by simplifications with node voltages as unknowns, plus a third form using currents as unknowns to support that approach. Node equations are written with currents positive into each node (slide 4).

Ohm's Law:

$$\frac{V_S - V_1}{R_A} = i_A$$

$$\frac{V_S - V_2}{R_C} = i_C$$

$$\frac{V_1}{R_B} = i_B$$

$$\frac{V_2}{R_D} = i_D$$

Loop 1:

$$(0 - V_2) + K_{IV} \cdot i_C + (V_1 - 0) = 0$$

$$-V_2 + K_{IV} \cdot \frac{V_S - V_2}{R_C} + V_1 = 0$$

$$-i_D \cdot R_D + K_{IV} \cdot i_C + i_B \cdot R_B = 0$$

Loop 2:

$$(0 - K_{IV} \cdot i_C) + (V_2 - V_S) + (V_S - V_1) = 0$$

$$-K_{IV} \cdot \frac{V_S - V_2}{R_C} + V_2 - V_1 = 0$$

$$-K_{IV} \cdot i_C - i_C \cdot R_C + i_A \cdot R_A = 0$$

Loop 3:

$$(0 - V_S) + (V_S - V_2) + (V_2 - 0) = 0$$

$$-V_S + i_C \cdot R_C + i_D \cdot R_D = 0$$

Node 1:

$$i_A - i_{CVS} - i_B = 0$$

$$\frac{V_S - V_1}{R_A} - i_{CVS} - \frac{V_1}{R_B} = 0$$

Node 2:

$$i_C + i_{CVS} - i_D = 0$$

$$\frac{V_S - V_2}{R_C} + i_{CVS} - \frac{V_2}{R_D} = 0$$

Node 3 (supernode with ground):

$$i_B + i_D - i_A - i_C = 0$$
$$\frac{V_1}{R_B} + \frac{V_2}{R_D} - \frac{V_S - V_1}{R_A} - \frac{V_S - V_2}{R_C} = 0$$

Examining these equations, we see that simple equations with unknowns as the five currents through the resistors and the controlled voltage source are possible using the three loop equations and the first two node equations; the Node 3 equation is the sum of the first two and is redundant with the first two. Thus we have five equations and five unknowns.

$$\begin{bmatrix} 0 & R_B & K_{IV} & -R_D & 0 \\ R_A & 0 & -R_C - K_{IV} & 0 & 0 \\ 0 & 0 & R_C & R_D & 0 \\ 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \\ i_{CVS} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_S \\ 0 \\ 0 \end{bmatrix}$$

Examining these same equations, we see that we can use the Loop1 and Node 3 equations to write two equations in V_1 and V_2. We write this equation

$$\begin{bmatrix} 1 & -\left(1 + \frac{K_{IV}}{R_C}\right) \\ \left(\frac{1}{R_A} + \frac{1}{R_B}\right) & \left(\frac{1}{R_C} + \frac{1}{R_D}\right) \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{K_{IV}}{R_C} \cdot V_S \\ -\left(\frac{1}{R_A} + \frac{1}{R_C}\right) \cdot V_S \end{bmatrix}.$$

The solution to this equation is

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & -\left(1 + \frac{K_{IV}}{R_C}\right) \\ \left(\frac{1}{R_A} + \frac{1}{R_B}\right) & \left(\frac{1}{R_C} + \frac{1}{R_D}\right) \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\frac{K_{IV}}{R_C} \cdot V_S \\ \left(\frac{1}{R_A} + \frac{1}{R_C}\right) \cdot V_S \end{bmatrix}$$

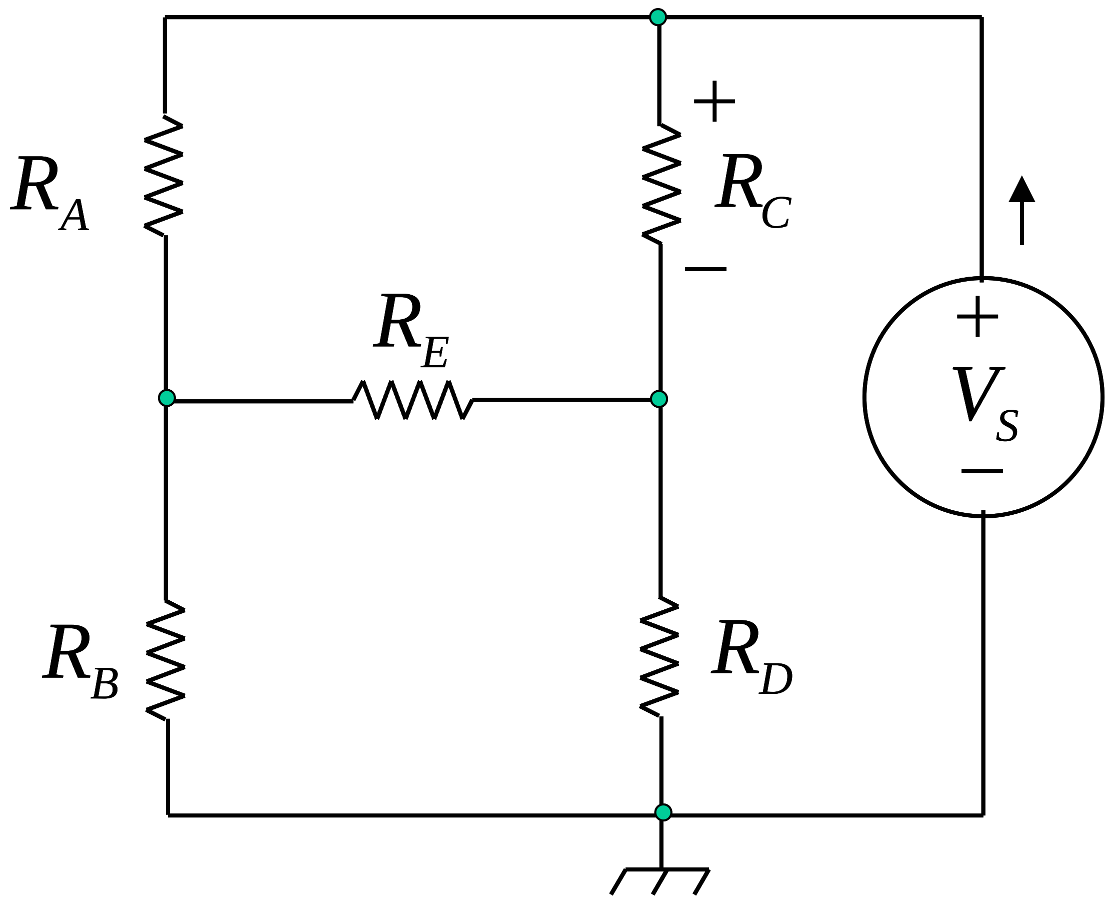
$$= \frac{1}{\left(\frac{1}{R_C} + \frac{1}{R_D}\right) + \left(\frac{1}{R_A} + \frac{1}{R_B}\right) \cdot \left(1 + \frac{K_{IV}}{R_C}\right)} \begin{bmatrix} \left(\frac{1}{R_C} + \frac{1}{R_D}\right) & \left(1 + \frac{K_{IV}}{R_C}\right) \\ -\left(\frac{1}{R_A} + \frac{1}{R_B}\right) & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{K_{IV}}{R_C} \cdot V_S \\ \left(\frac{1}{R_A} + \frac{1}{R_C}\right) \cdot V_S \end{bmatrix}$$

The currents are found by Ohm's law except i_{CVS}, which is found from the Node 1 or Node 2 current equations. The solution for the circuit values given is

$$V_1 = 6.049\,V$$
$$V_2 = 6.645\,V$$
$$i_A = 0.595\ A$$
$$i_B = 0.302\ A$$
$$i_C = 0.138\ A$$
$$i_D = 0.431\ A$$
$$i_{CVS} = 0.293\ A$$

Problem 4 (25%)

Find the value of R_D in the circuit below, with the other parameters given below the figure, so that the voltage across R_E is zero. Is this a standard value?



$$V_S = 15\,V \qquad R_A = 10\,k\Omega \qquad R_B = 22\,k\Omega$$

$$R_C = 20\,k\Omega \qquad R_D = ? \qquad R_E = 5\,k\Omega$$

Hint: The TI-89 EEPro application has a standard component value utility in its reference library.

Solution

This problem was inspired by Problem 4.3-7 page 143 and the Strain Gauge Bridge Design Example on page 183. As presented in here as Problem 4, the problem was simplified to present the question of how to equalize the voltage output of two voltage dividers.

The current through R_E is zero when the voltage divider formed by R_A and R_B produces the same voltage on the left side of R_E as the voltage divider formed by R_C and R_D produces on the right side of R_E. The two voltage dividers are separable under the condition that the voltage across R_E is zero because the current through R_E will be zero. The condition for the voltage dividers producing an equal voltage is

$$\frac{R_B}{R_A + R_B} = \frac{R_D}{R_C + R_D}.$$

Taking the reciprocal of both sides of this equation, we have

$$1 + \frac{R_A}{R_B} = 1 + \frac{R_C}{R_D}$$

or,

$$R_D = \frac{R_B \cdot R_C}{R_A} = 44\ k\Omega.$$

Since many of you don't have EEPro on your TI-89 calculators, and you aren't ECE's, I weighted the "standard value" part of this question as zero.