Networks I for M.E.
ECE 09.201 - 2
Final Examination October 23, 2006
SOLUTIONS

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Problem 1 (25%)

Part A (25%)

The total charge that has entered a circuit element is

$$q(t)\begin{cases} = q_0 \cdot \left(1 - \exp\left(-\dfrac{t}{T_C}\right)\right), \ t \geq 0 \\ = 0, \ t < 0 \end{cases}$$

Determine the current in this circuit element for $t \geq 0$. There are no numbers given for the initial charge q_0 or the time constant T_C. Write the equation.

Part B (25%)

A battery is starting a car on a cold morning. The starter draws 1000 A of current for 3 seconds before the car starts. The battery maintains 10 V across the starter during this time.

- What is the total charge transferred through the starter to start the car?
- What is the power at the starter?
- What is the total energy that is used to start the car?

Part C (25%)

Find the voltage from B to A in the circuit to the right with the parameters

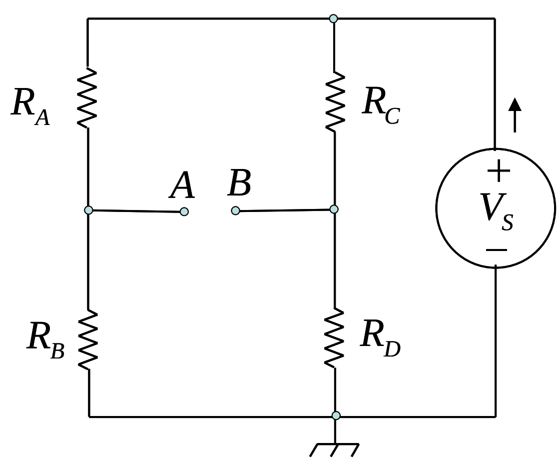
$V_S = 15\,V$

$R_A = 1\,k\Omega$ (Note that $1\,k\Omega = 1{,}000\Omega$)

$R_B = 2\,k\Omega$

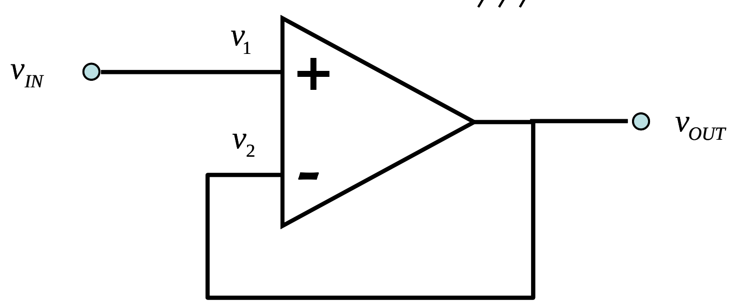
$R_C = 4\,k\Omega$

$R_D = 8\,k\Omega$



Part D (25%)

In the op-amp circuit to the lower right, assume that the op-amp is perfect. If $v_{IN} = +2\,V$ what are v_1, v_2 and e_{OUT}? Why? You may refer to the textbook in your reply.



Problem 1 Solution

Part A Solution

The current is the charge flow rate, which is computed from the equation of the charge entering a circuit versus time as the derivative with respect to time.

$$\frac{d}{dt}q(t)\begin{cases}=\dfrac{q_0}{T_C}\cdot\exp\left(-\dfrac{t}{T_C}\right),\ t\geq 0\\ =0,\ t<0\end{cases}$$

Part B Solution

The charge is the current times time, if the current is constant as the problem statement stipulates. Thus the total charge in Coulombs is

$$q_{TOTAL}=(1000\ A)\cdot(3\ s)=3000\ C\ .$$

The power at the starter in Watts for 1,000 A and 10 volts, as given in the problem statement, is

$$p_{STARTER}=(1000\ A)\cdot(10\ V)=10,000\ W$$

The total energy in Joules is the power in Watts times the time in seconds, or

$$e_{STARTINT}=(10,000\ W)\cdot(3\ s)=30,000\ J$$

Note that at 746 Watts per horsepower, the power into the starter, not out of the starter, is 13.4 hp. If the starter efficiency is about 80%, this is about 10 hp.

Part C Solution

Resistors R_A and R_B form a voltage divider between V_S and ground for point *A* in the circuit. Likewise, resistors R_C and R_D form a voltage divider between V_S and ground for point *B* in the circuit. Both of these voltage dividers produce 1/3 of V_S, so the voltage between points *A* and *B* in the circuit is zero Volts.

Part D Solution

There is a direct connection between v_{IN} and v_1. Likewise there is a direct connection between v_{OUT} and v_2. The perfect op-amp will have the operating conditions given in Table 6.3-1 on the top of page 204 of the text, specifically that the voltages at the input terminals will be equal; that is,

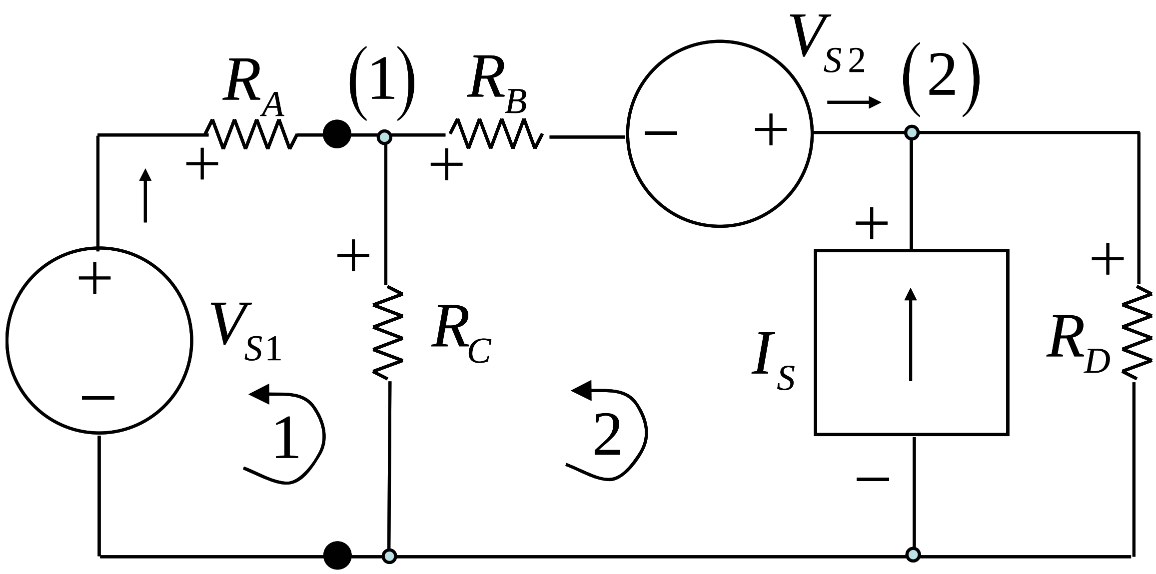
$$v_2-v_1=0\ .$$

This all the voltages in the circuit are equal to v_{IN}, or +2 Volts.

This circuit is treated in Example .3-1 pages 204-205.

Problem 2 (25%)

For the circuit shown below, write a set of Kirchhoff's voltage and current laws. Rearrange the equations that you select into matrix format. ***You do not have to invert the matrix.*** However, if the matrix that you select is singular, i.e. some of the rows can be found by adding and subtracting other rows, you will not be given full credit. Do not use numerical values in the solution that you present for this problem. Use the format in the example in Slide 8 of the handout – i.e. label the node and loop equations, and the columns of the matrix with the unknowns..



Problem 2 Solution, node voltage notation

Note that node and loop labels and "+" signs on the resistors have been added. We note that the left terminal of R_A is a supernode with ground, and the right terminal of R_B is a supernode with Node 2. A third loop is trivial as it simply equations the voltage drop across R_D with V_2 which is also the voltage across the current source. We will begin with Ohm's law, then give the loop and node equations.

$$\frac{V_{S1}-V_1}{R_A}=i_A$$

$$\frac{V_1-(V_2-V_{S2})}{R_B}=i_B$$

$$\frac{V_1}{R_C}=i_C$$

$$\frac{V_2}{R_D}=i_D$$

$$\text{Loop 1:} \quad (0 - V_1) + (V_1 - V_{S1}) + (V_{S1} - 0) = 0$$
$$\text{Loop 2:} \quad (0 - V_2) + V_{S2} - i_B \cdot R_B + V_1 = 0$$
$$\text{Node 1:} \quad +i_A - i_B - i_C = 0$$
$$\text{Node 2:} \quad +i_B + I_S - i_D = 0$$

Now, we see that the Node 1 equation can be used to use $(i_A - i_B)$ in place of the unknown i_C, and the Node 2 equation couples the unknowns i_B and i_D as differing by I_S, so that only one need be taken as an unknown. Thus the circuit is completely characterized by either the node voltages V_1 and V_2 or by the currents i_A and i_B. We have a choice of using the Ohm's Law equations in the loop or node equations to give us a matrix equation for the two remaining voltage or current unknowns. We use Ohm's law in the node equations to pose them in terms of the node voltages,

$$\text{Node 1:} \quad \frac{V_{S1} - V_1}{R_A} - \frac{V_1 + V_{S2} - V_2}{R_B} - \frac{V_1}{R_C} = 0$$
$$\text{Node 2:} \quad \frac{V_1 + V_{S2} - V_2}{R_B} + I_S - \frac{V_2}{R_D} = 0.$$

In matrix form, these equations are

$$\begin{matrix} \\ \text{Node 1} \\ \\ \text{Node 2} \end{matrix} \quad \begin{matrix} V_1 & & V_2 \\ \end{matrix}$$
$$\begin{bmatrix} -\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) & \frac{1}{R_B} \\ \frac{1}{R_B} & -\left(\frac{1}{R_B} + \frac{1}{R_D}\right) \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{V_{S1}}{R_A} + \frac{V_{S2}}{R_B} \\ -\frac{V_{S2}}{R_B} - I_S \end{bmatrix}.$$

In terms of the currents i_A and i_B we have the loop equations as

$$\text{Loop 1:} \quad -(i_A - i_B) \cdot R_C - i_A \cdot R_A + V_{S1} = 0$$
$$\text{Loop 2:} \quad -(i_B + I_S) \cdot R_D + V_{S2} - i_B \cdot R_B + (i_A - i_B) \cdot R_C = 0$$

With this form, we can write the matrix equations for of the currents i_A and i_B as

$$\begin{matrix} \text{Loop 1} \\ \text{Loop 2} \end{matrix} \quad \begin{matrix} i_A \quad\quad\quad\quad i_B \end{matrix}$$
$$\begin{bmatrix} -(R_A + R_C) & R_C \\ R_C & -(R_B + R_C + R_D) \end{bmatrix} \cdot \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -V_{S1} \\ -V_{S2} + I_S \cdot R_D \end{bmatrix}.$$

Please see the solution for Problem 3 following for numerical values of the resistors and sources, of the voltages and currents in that specific circuit.

Problem 2 solution, currents as unknowns

We will begin with Ohm's law, then give the loop and node equations.

$$i_A = \frac{V_A}{R_A}$$

$$i_B = \frac{V_B}{R_B}$$

$$i_C = \frac{V_C}{R_C}$$

$$i_D = \frac{V_D}{R_D}$$

Loop 1: $-V_C - V_A + V_{S1} = 0$

Loop 2: $-V_S + V_{S2} - V_B + V_C = 0$

Node 1: $i_A - i_B - i_C = 0$

Node 2: $i_B + I_S - i_D = 0$

We note that the voltage V_S across the current source is V_D, which can also be inserted by the use of a third loop incorporating only the current source and R_D. We can obtain four equations in the four unknown currents through the resistors by using Ohm's law in the loop equations and combining with the node equations to get four equations in the currents through the resistors.

Loop 1: $-i_C \cdot R_C - i_A \cdot R_A + V_{S1} = 0$

Loop 2: $-i_D \cdot R_D + V_{S2} - i_B \cdot R_B + i_C \cdot R_C = 0$

Collecting the loop and node equations as a vector-matrix equation, we have

$$\begin{array}{c} \\ \text{Loop 1} \\ \text{Loop 2} \\ \text{Node 1} \\ \text{Node 2} \end{array} \begin{array}{c} \begin{array}{cccc} i_A & i_B & i_C & i_D \end{array} \\ \begin{bmatrix} -R_A & 0 & -R_C & 0 \\ 0 & -R_B & R_C & -R_D \\ 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \end{array} \cdot \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} -V_{S1} \\ -V_{S2} \\ 0 \\ -I_S \end{bmatrix}$$

Problem 2 solution, voltages as unknowns

We use Ohm's law in the node equations to pose them in terms of the voltage drops across the resistors and collect as a matrix equation.

Node 1: $\frac{V_A}{R_A} - \frac{V_B}{R_B} - \frac{V_C}{R_C} = 0$

Node 2: $\frac{V_B}{R_B} + I_S - \frac{V_D}{R_D} = 0$

$$\begin{array}{l} \\ \text{Loop 1} \\ \text{Loop 2} \\ \text{Node 1} \\ \text{Node 2} \end{array} \begin{array}{c} \begin{array}{cccc} V_A & V_B & V_C & V_D \end{array} \\ \begin{bmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & -1 \\ \frac{1}{R_A} & -\frac{1}{R_b} & -\frac{1}{R_C} & 0 \\ 0 & \frac{1}{R_B} & 0 & -\frac{1}{R_D} \end{bmatrix} \end{array} \cdot \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -V_{S1} \\ -V_{S2} \\ 0 \\ -I_S \end{bmatrix}$$

Algebraic solution of circuit

From any one of the above solutions, we have an algebraic solution of the circuit. The determinant of any valid matrix that solves the circuit is, within a multiplicative factor,

$$D = R_A \cdot \left(R_B + R_C + R_D\right) + R_C \cdot \left(R_B + R_D\right).$$

The currents are given by

$$\begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \frac{1}{D} \cdot \begin{bmatrix} R_B + R_C + R_D & R_C & R_C \cdot \left(R_B + R_D\right) & -R_C \cdot R_D \\ R_C & R_A + R_C & -R_A \cdot R_C & -R_D \cdot \left(R_A + R_C\right) \\ R_B + R_D & -R_A & -R_A \cdot \left(R_B + R_D\right) & R_A \cdot R_D \\ R_C & R_A + R_C & -R_A \cdot R_C & R_A \cdot \left(R_B + R_C\right) + R_B \cdot R_C \end{bmatrix} \cdot \begin{bmatrix} V_{S1} \\ V_{S2} \\ 0 \\ I_S \end{bmatrix}.$$

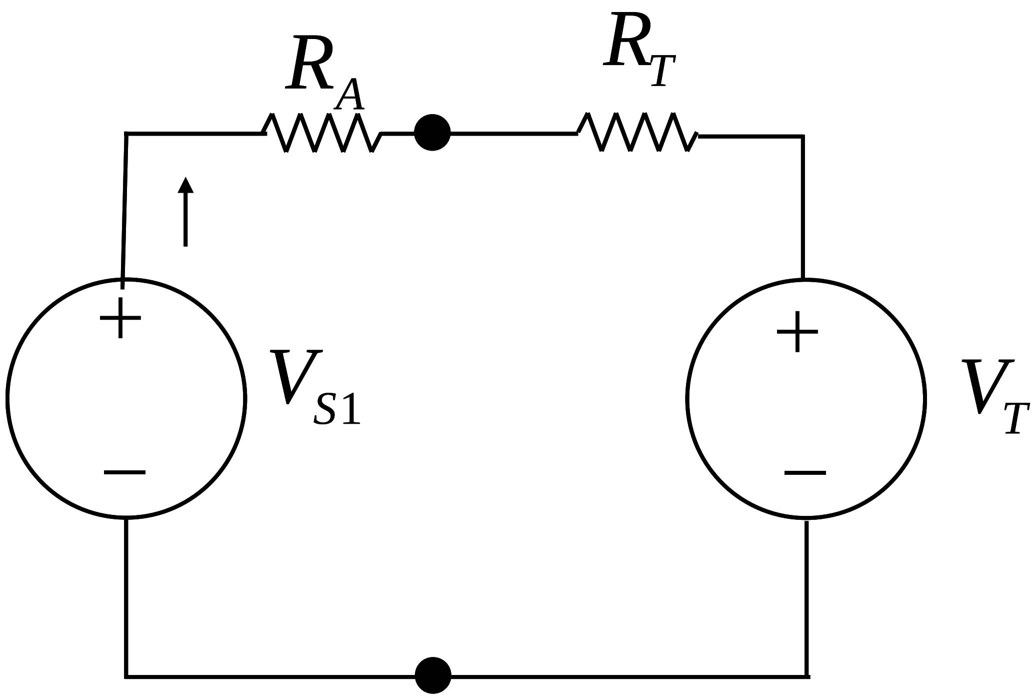
Note that the first, second and third rows had their signs reversed prior to inverting the matrix. The node voltages are

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{D} \cdot \begin{bmatrix} R_A \cdot R_C \cdot \left(R_B + R_D\right) & R_A \cdot R_C \cdot R_D \\ R_A \cdot R_C \cdot R_D & R_D \cdot \left(R_A \cdot \left(R_B + R_C\right) + R_B \cdot R_C\right) \end{bmatrix} \cdot \begin{bmatrix} \frac{V_{S1}}{R_A} - \frac{V_{S2}}{R_B} \\ \frac{V_{S2}}{R_B} + I_S \end{bmatrix}.$$

Note that the matrix was negated prior to inversion, and the constant factor in its determinant was absorbed in the matrix to allow the use of the same equation for the determinant in the solution. These solutions are helpful in looking at solutions by superposition.

Problem 3 (25%)

For the circuit of Problem 2, find V_T and R_T in the equivalent circuit below. Use the numerical values given below the figure for the source voltages and current, and for the resistances. Then, find the current through the voltage source V_{S1} using the sign convention given in the diagram.



$V_{S1} = 15\,V \quad V_{S2} = 5\,V \quad I_S = 2\,mA$

$R_A = 10\,k\Omega \quad R_B = 20\,k\Omega \quad R_C = 5\,k\Omega$

$R_D = 5\,k\Omega$

Note that $1\,mA = 0.001\,A$ and $1\,k\Omega = 1,000\Omega$.

Problem 3 Solution, Thevenin and Norton equivalent circuits

We will begin with the circuit of Problem 2. Note that the Thevenin equivalent circuit to the right of the black dots can be found by any method that finds the open circuit voltage and the closed circuit current or the resistance between the black dots, due only to the circuit to the right of the black dots, with the sources set to zero. These methods include superposition and matrix formulation of Kirchhoff's laws. The solution given here uses successive transformations back and forth from Thévenin and Norton equivalent circuits.

We see that the combination of I_S and R_D is a Norton equivalent circuit, so we make the transformation given in the handout, Slide 10, which summarizes the basic principles underlying Sections 5.4 and 5.5 of the text. We show that the circuit to the right of Node

2 is, including the current source, a resistance of R_D in series with a voltage source of magnitude $I_S \cdot R_D$.

We now see that we can combine R_B and the source V_{S2} to form a Thévenin equivalent circuit with a resistance of $\left(R_B + R_D\right)$ and a voltage of $\left(I_S \cdot R_D - V_{S2}\right)$. The next step is to convert this into a Norton equivalent circuit, with a parallel resistance of $\left(R_B + R_D\right)$ and a current source of magnitude

$$I_{SE} = \frac{I_S \cdot R_D - V_{S2}}{R_B + R_D}$$

in parallel with a resistance of $\left(R_B + R_D\right)$.

We now note that, in the circuit of Problem 2, to the right of the large black dots we have R_C in parallel with $\left(R_B + R_D\right)$ and a current source of magnitude I_{SE}. We find the resistance of R_C in parallel with $\left(R_B + R_D\right)$ as the final Thévenin equivalent resistance,

$$R_T = \frac{1}{\frac{1}{R_C} + \frac{1}{R_B + R_D}}.$$

We use this resistance with the Norton equivalent current source of I_{SE} to find the final Thévenin equivalent voltage as

$$V_T = I_{SE} \cdot R_T = \left(I_S \cdot R_D - V_{S2}\right) \cdot \frac{R_C}{R_B + R_C + R_D}.$$

Both equations can be written by inspection using the short circuit current and open circuit voltage method, or the methods of Example 5.4-2 on pages 165 and 166, once the first transformation is done.

Finally, the current i_{VS1} through the voltage source V_{S1} is given as

$$i_{VS1} = \frac{V_{S1} - V_T}{R_A - R_T}.$$

With the values given in the problem statement, we have the numerical values

$$I_S \cdot R_D = 10\,V$$
$$I_{SE} = \frac{10\,V - 5\,V}{20\,k\Omega + 5\,k\Omega} = 0.2\,mA$$
$$R_T = \frac{1}{\frac{1}{5\,k\Omega} + \frac{1}{25\,k\Omega}} = \frac{1}{0.24}\,k\Omega = 4.1667\,k\Omega$$
$$V_T = 5\,V \cdot \frac{5\,k\Omega}{30\,k\Omega} = \frac{5}{6}\,V = 0.8333\,V$$
$$i_{VS1} = \frac{15\,V - 0.8333\,V}{10\,k\Omega + 4.1667\,k\Omega} = \frac{14.1667\,V}{14.1667\,k\Omega} = 1\,mA.$$

Numerical solution for the entire circuit of problems 2 and 3

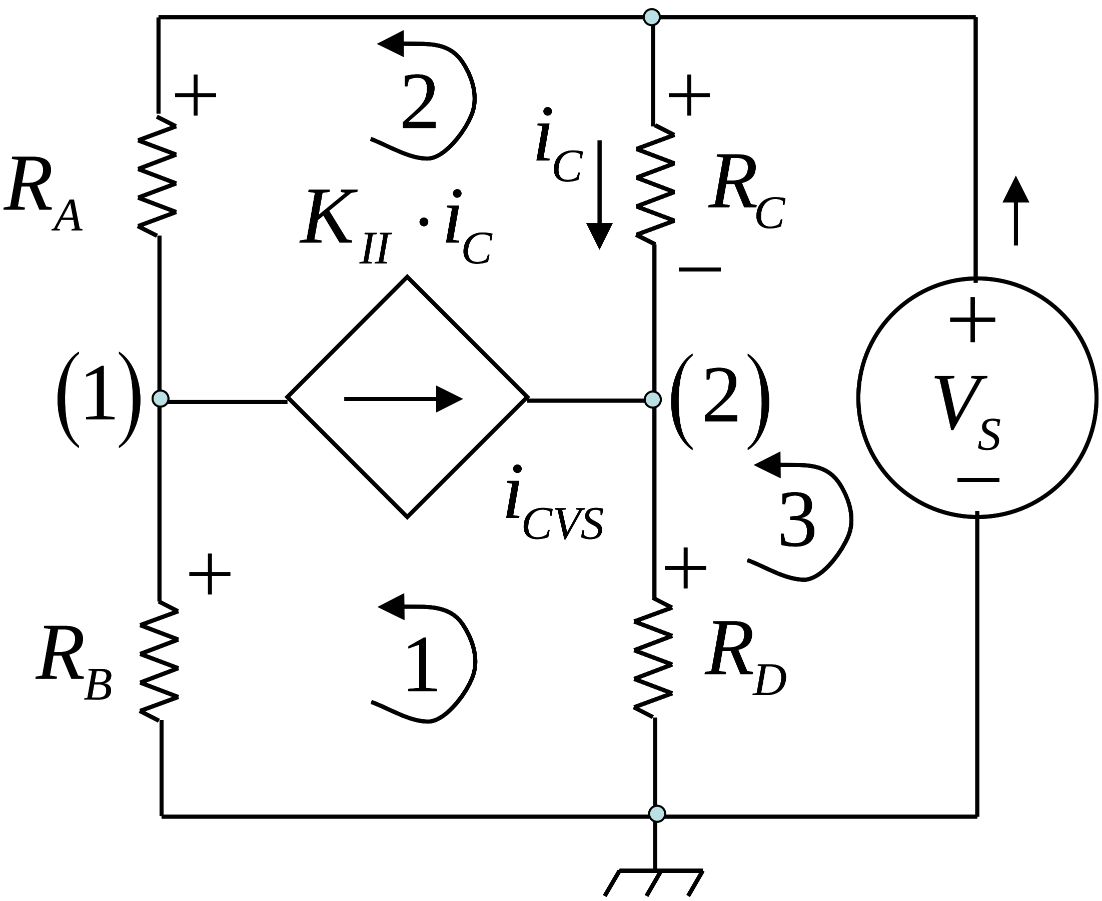
Note that, for the values given in problem 3, the complete solution to the circuit is

$$V_1 = 5\,V$$
$$V_2 = 10\,V$$
$$i_A = 1\,mA$$
$$i_B = 0\,mA$$
$$i_C = 1\,mA$$
$$i_D = 2\,mA.$$

Thus, for the values given in the problem statement, there is no current through R_B or the voltage source V_{S2}.

Problem 4 (25%)

Solve the circuit in the figure for the parameters given below the figure. Give solutions for the node voltages and the currents through all the resistors. Note that the controlled source is a current source controlled by a current.



$$V_S = 15\,V \qquad K_{II} = 4 \qquad R_A = 10\,k\Omega$$

$$R_B = 15\,k\Omega \quad R_C = 30\,k\Omega \quad R_D = 12\,k\Omega$$

Note that $1\,k\Omega = 1{,}000\,\Omega$.

Hint A: Look for parts of the circuit that operate without dependence on other parts.

Hint B: The solution of the circuit is completely characterized by the voltages on either side of the controlled current source.

Problem 4 solution, direct method

Except for the values given in the problem statement, this is the same as Quiz 2, Problem 3, which many of you had a problem with. This Final Examination problem allows you to take advantage of the review of Quiz 2 and get credit for it.

We will not ignore the hints. We will write the Ohm's law and Kirchhoff's law equations, then examine them in response to Hint A. Hint B simply points out that finding V_1 and V_2, with Ohm's law, provides a complete solution for the circuit. We begin with Ohm's law, then give the loop and node equations.

$$\frac{V_S - V_1}{R_A} = i_A$$

$$\frac{V_1}{R_B} = i_B$$

$$\frac{V_S - V_2}{R_C} = i_C$$

$$\frac{V_2}{R_D} = i_D$$

Loop 1: $\left(0 - V_2\right) + \left(V_2 - V_1\right) + V_1 = 0$

Loop 2: $\left(V_1 - V_2\right) + \left(V_2 - V_S\right) + \left(V_S - V_1\right) = 0$

Loop 3: $\left(0 - V_S\right) + \left(V_S - V_2\right) + V_2 = 0$

Node 1: $i_A - i_B - K_{ii} \cdot i_C = 0$

Node 2: $K_{ii} \cdot i_C + i_C - i_D = 0$

We see that the Node 2 equation gives i_D as a function of i_C, which with the Loop 3 equation and Ohm's law for R_C and R_D provides a way to solve for i_C. From that the solution for the rest of the circuit follows from the other equations. Taking this route, we have

$$i_D = \left(K_{II} + 1\right) \cdot i_C$$

combined with the Loop 3 equation and Ohm's law for R_C and R_D,

$$-V_S + i_C \cdot R_C + i_D \cdot R_D = -V_S + i_C \cdot R_C + \left(K_{II} + 1\right) \cdot i_C \cdot R_D = 0$$

from which we have

$$i_C = \frac{V_S}{R_C + \left(K_{II} + 1\right) \cdot R_D}.$$

The rest of the circuit follows directly. We use Ohm's law for R_D to find

$$V_2 = \frac{\left(K_{II} + 1\right) \cdot R_D}{R_C + \left(K_{II} + 1\right) \cdot R_D} \cdot V_S.$$

We use Ohm's law for R_A and R_B in the Node 1 equation to find V_1,

$$\frac{V_S - V_1}{R_A} - \frac{V_1}{R_B} - K_{II} \cdot i_C = 0$$

or,

$$\left(\frac{1}{R_A} + \frac{1}{R_B}\right) \cdot V_1 = \frac{V_S}{R_A} - K_{II} \cdot i_C$$

so that

$$V_1 = \frac{\frac{V_S}{R_A} - K_{II} \cdot i_C}{\frac{1}{R_A} + \frac{1}{R_B}} = \frac{R_B}{R_A + R_B} \cdot \frac{-K_{II} \cdot R_A + R_C + (1 + K_{II}) \cdot R_D}{R_C + (1 + K_{II}) \cdot R_D} \cdot V_S$$

and the rest of the circuit is solved with Ohm's law.

Problem 4 solution, node voltage notation

For those of you who solved this equation by the method of using Kirchhoff's loop and node laws in matrix form, a simple method is to define unknowns as the voltages across the resistors and the current source, plus the currents in the resistors, and write all the Ohm's law and Kirchhoff's laws equations in a nine by nine matrix format. Allowing duplicate variables can produce an 11 by 11 matrix and some Final Examination responses did that, and did it correctly. That is, of course, acceptable and even desirable when Matlab or similar capability is available for numerical solution of the circuit, and can be used with calculators such as the TI-89. First, we will use voltage node notation and the current node equations, which will provide a two by two matrix solution that is more amenable to finding equations, or hand computation.

As before, we use Ohm's law with the Node 1 and Node 2 equations:

$$\text{Node 1:} \quad \frac{V_S - V_1}{R_A} - \frac{V_1}{R_B} - K_{II} \cdot \frac{V_S - V_2}{R_C} = 0$$

$$\text{Node 2:} \quad (K_{II} + 1) \cdot \frac{V_S - V_2}{R_C} - \frac{V_2}{R_D} = 0$$

The matrix form of these equations is

$$\begin{array}{c} \\ \text{Node 1} \\ \\ \text{Node 2} \end{array} \begin{array}{c} \begin{array}{cc} V_1 \qquad\qquad & V_2 \end{array} \\ \begin{bmatrix} -\left(\frac{1}{R_A} + \frac{1}{R_B}\right) & \frac{K_{II}}{R_C} \\ 0 & -\left(\frac{K_{II} + 1}{R_C} + \frac{1}{R_D}\right) \end{bmatrix} \end{array} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{V_S}{R_A} + \frac{K_{II} \cdot V_S}{R_C} \\ -\frac{(K_{II} + 1) \cdot V_S}{R_C} \end{bmatrix}$$

Note that the bottom row tells us that V_2 can be solved directly, and that the solution is independent of V_1. In fact, this gives us the same equation that we have for V_2 above using the direct method.

Problem 4 solution, currents as unknowns

For those who solve for currents, we have a minimum of three unknowns. We can use the Node 2 equation to substitute for i_D or sum the two node equations to find

$$i_A - i_B + i_C - i_D = 0$$

and use this equation to eliminate one of the currents, as examples. For simplicity in the equations and matrix, we will use all four currents as unknowns. Our equations will be the three loop equations with Ohm's law used to put the voltage drops in terms of the currents, and either of the two node equations. We will use Loop 3 because it does not contain $(V_1 - V_2)$ as an unknown, and sum Loops 1, 2 and 3 to obtain a second loop equation that does not contain $(V_1 - V_2)$ as an unknown. These equations are

$$\begin{aligned}
\text{Loop 1+2+3:}\quad & -V_S + i_A \cdot R_A + i_B \cdot R_B = 0 \\
\text{Loop 3:}\quad & -V_S + i_C \cdot R_C + i_D \cdot R_D = 0 \\
\text{Node 1:}\quad & i_A - i_B - K_{ii} \cdot i_C = 0 \\
\text{Node 2:}\quad & K_{ii} \cdot i_C + i_C - i_D = 0.
\end{aligned}$$

These equations are, in matrix form,

$$\begin{matrix} \text{Loop 1+2+3} \\ \text{Loop 3} \\ \text{Node 1} \\ \text{Node 2} \end{matrix}
\overset{\begin{matrix} i_A & i_B & i_C & i_D \end{matrix}}{\begin{bmatrix} R_A & R_B & 0 & 0 \\ 0 & 0 & R_C & R_D \\ 1 & -1 & -K_{II} & 0 \\ 0 & 0 & (K_{II}+1) & -1 \end{bmatrix}} \cdot \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} V_S \\ V_S \\ 0 \\ 0 \end{bmatrix}.$$

Solution of the circuit of problem 4

Algebraic solution

From the node voltage solution, we have the voltages at the two unknown nodes aas

$$V_1 = \frac{R_B}{R_A + R_B} \cdot \frac{-K_{II} \cdot R_A + R_C + (1 + K_{II}) \cdot R_D}{R_C + (1 + K_{II}) \cdot R_D} \cdot V_S$$

$$V_2 = \frac{(1 + K_{II}) \cdot R_D}{R_C + (1 + K_{II}) \cdot R_D} \cdot V_S$$

The resistor voltages can be found from these by subtraction and the resistor currents found by Ohm's law. Explicit equations for the resistor currents are found from the second solution as

$$i_A = \frac{K_{II} \cdot R_B + R_C + (1 + K_{II}) \cdot R_D}{(R_A + R_B) \cdot (R_C + (1 + K_{II}) \cdot R_D)} \cdot V_S$$

$$i_B = \frac{-K_{II} \cdot R_A + R_C + (1 + K_{II}) \cdot R_D}{(R_A + R_B) \cdot (R_C + (1 + K_{II}) \cdot R_D)} \cdot V_S$$

$$i_C = \frac{V_S}{R_C + (1 + K_{II}) \cdot R_D}$$

$$i_D = \frac{(1 + K_{II}) \cdot V_S}{R_C + (1 + K_{II}) \cdot R_D}$$

The equations for the voltages across the resistors are the currents times the values of the respective resistances.

Note that the values of V_2, i_C and i_D are independent of R_A and R_B.

Numerical solution

The solution of the circuit is found from any of the methods and the component values given in problem 3 is

$$V_1 = 5\,V$$
$$V_2 = 10\,V$$
$$i_A = 1\,mA$$
$$i_B = \tfrac{1}{3}\,mA$$
$$i_C = \tfrac{1}{6}\,mA$$
$$i_D = \tfrac{5}{6}\,mA$$