The Prolem Solving Method using Kirchoff's Laws

Loop 1

$-v_s + v_a + v_c = 0$

Loop 2

$-v_c + v_b + v_{is} = 0$

Node 1

$i_{vs} - i_a = 0$

Node 2

$i_a - i_c - i_b = 0$

Node 3

$i_b + i_s = 0$

Node 4 is redundant with Node 1, Node 2, and Node 3 (Why?)

$-i_{vs} + i_c - i_s = 0$

Ohm's Law for R.a

$v_a = i_a \cdot R_a$

Ohm's Law for R.b

$v_b = i_b \cdot R_b$

Ohm's Law for R.c

$v_c = i_c \cdot R_c$

Sources are v.s and i.s
Unknowns are i.vs, v.is, and the voltages and currents across the resistors.

Arranging the equations as a vector-matrix equation:

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -R_a & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -R_b & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -R_c \end{pmatrix} \cdot \begin{pmatrix} i_{vs} \\ v_{is} \\ v_a \\ v_b \\ v_c \\ i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} v_s. \\ 0 \\ 0 \\ 0 \\ i_s \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The solution, using matrix notation:

$$\mathrm{soln}(v_s, i_s, R_a, R_b, R_c) := \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -R_a & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -R_b & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -R_c \end{pmatrix}^{-1} \cdot \begin{pmatrix} v_s \\ 0 \\ 0 \\ 0 \\ i_s \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathrm{soln}(12, .01, 10000, 22000, 12000) = \begin{pmatrix} -4.909 \times 10^{-3} \\ 281.091 \\ -49.091 \\ -220 \\ 61.091 \\ -4.909 \times 10^{-3} \\ -0.01 \\ 5.091 \times 10^{-3} \end{pmatrix}$$

For trial values as given

$$\mathrm{soln}(12, .001, 10000, 22000, 12000) = \begin{pmatrix} 0 \\ 34 \\ 0 \\ -22 \\ 12 \\ 0 \\ -1 \times 10^{-3} \\ 1 \times 10^{-3} \end{pmatrix}$$

Change i.s to 1 ma

An algebraic solution

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -R_a & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -R_b & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -R_c \end{pmatrix}$$

by matrix inversion, yields

[OMITTED LARGE EQUATION]

simplifies to

$$
\begin{bmatrix}
\frac{1}{R_c + R_a} & 0 & 1 & \frac{R_c}{R_c + R_a} & \frac{-R_c}{R_c + R_a} & \frac{-1}{R_c + R_a} & 0 & \frac{-1}{R_c + R_a} \\
\frac{R_c}{R_c + R_a} & 1 & 0 & (-R_a)\cdot\frac{R_c}{R_c + R_a} & \frac{R_c\cdot R_b + R_a\cdot R_c + R_a\cdot R_b}{R_c + R_a} & \frac{-R_c}{R_c + R_a} & -1 & \frac{R_a}{R_c + R_a} \\
\frac{R_a}{R_c + R_a} & 0 & 0 & R_a\cdot\frac{R_c}{R_c + R_a} & (-R_a)\cdot\frac{R_c}{R_c + R_a} & \frac{R_c}{R_c + R_a} & 0 & \frac{-R_a}{R_c + R_a} \\
0 & 0 & 0 & 0 & -R_b & 0 & 1 & 0 \\
\frac{R_c}{R_c + R_a} & 0 & 0 & (-R_a)\cdot\frac{R_c}{R_c + R_a} & R_a\cdot\frac{R_c}{R_c + R_a} & \frac{-R_c}{R_c + R_a} & 0 & \frac{R_a}{R_c + R_a} \\
\frac{1}{R_c + R_a} & 0 & 0 & \frac{R_c}{R_c + R_a} & \frac{-R_c}{R_c + R_a} & \frac{-1}{R_c + R_a} & 0 & \frac{-1}{R_c + R_a} \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\frac{1}{R_c + R_a} & 0 & 0 & \frac{-R_a}{R_c + R_a} & \frac{R_a}{R_c + R_a} & \frac{-1}{R_c + R_a} & 0 & \frac{-1}{R_c + R_a}
\end{bmatrix}
\cdot\frac{R_c + R_a}{R_c\cdot R_a}
$$

simplifies to

$$
\begin{bmatrix}
\frac{1}{R_a\cdot R_c} & 0 & \frac{R_c + R_a}{R_a\cdot R_c} & \frac{1}{R_a} & \frac{-1}{R_a} & \frac{-1}{R_a\cdot R_c} & 0 & \frac{-1}{R_a\cdot R_c} \\
\frac{1}{R_a} & \frac{R_c + R_a}{R_a\cdot R_c} & 0 & -1 & \frac{R_c\cdot R_b + R_a\cdot R_c + R_a\cdot R_b}{R_a\cdot R_c} & \frac{-1}{R_a} & \frac{-(R_c + R_a)}{R_a\cdot R_c} & \frac{1}{R_c} \\
\frac{1}{R_c} & 0 & 0 & 1 & -1 & \frac{1}{R_a} & 0 & \frac{-1}{R_c} \\
0 & 0 & 0 & 0 & (-R_b)\cdot\frac{R_c + R_a}{R_a\cdot R_c} & 0 & \frac{R_c + R_a}{R_a\cdot R_c} & 0 \\
\frac{1}{R_a} & 0 & 0 & -1 & 1 & \frac{-1}{R_a} & 0 & \frac{1}{R_c} \\
\frac{1}{R_a\cdot R_c} & 0 & 0 & \frac{1}{R_a} & \frac{-1}{R_a} & \frac{-1}{R_a\cdot R_c} & 0 & \frac{-1}{R_a\cdot R_c} \\
0 & 0 & 0 & 0 & \frac{-(R_c + R_a)}{R_a\cdot R_c} & 0 & 0 & 0 \\
\frac{1}{R_a\cdot R_c} & 0 & 0 & \frac{-1}{R_c} & \frac{1}{R_c} & \frac{-1}{R_a\cdot R_c} & 0 & \frac{-1}{R_a\cdot R_c}
\end{bmatrix}
\cdot
\begin{pmatrix} v_s \\ 0 \\ 0 \\ 0 \\ i_s \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

simplifies to

$$\left[\begin{array}{c} \dfrac{-\left[\left(-v_s\right) + i_s \cdot R_c\right]}{R_a \cdot R_c} \\ \dfrac{v_s \cdot R_c + i_s \cdot R_c \cdot R_b + i_s \cdot R_a \cdot R_c + i_s \cdot R_a \cdot R_b}{R_a \cdot R_c} \\ \dfrac{-\left[\left(-v_s\right) + i_s \cdot R_c\right]}{R_c} \\ \left(-R_b\right) \cdot \dfrac{R_c + R_a}{R_a \cdot R_c} \cdot i_s \\ \dfrac{v_s + i_s \cdot R_a}{R_a} \\ \dfrac{-\left[\left(-v_s\right) + i_s \cdot R_c\right]}{R_a \cdot R_c} \\ \left[-\left(R_c + R_a\right)\right] \cdot \dfrac{i_s}{R_a \cdot R_c} \\ \dfrac{v_s + i_s \cdot R_a}{R_a \cdot R_c} \end{array}\right] \cdot \frac{R_c \cdot R_a}{R_c + R_a}$$

simplifies to

$$
\begin{bmatrix}
\dfrac{-\left[\left(-v_s\right) + i_s \cdot R_c\right]}{R_c + R_a} \\
\dfrac{v_s \cdot R_c + i_s \cdot R_c \cdot R_b + i_s \cdot R_a \cdot R_c + i_s \cdot R_a \cdot R_b}{R_c + R_a} \\
\left[-\left[\left(-v_s\right) + i_s \cdot R_c\right]\right] \cdot \dfrac{R_a}{R_c + R_a} \\
\left(-R_b\right) \cdot i_s \\
\left(v_s + i_s \cdot R_a\right) \cdot \dfrac{R_c}{R_c + R_a} \\
\dfrac{-\left[\left(-v_s\right) + i_s \cdot R_c\right]}{R_c + R_a} \\
-i_s \\
\dfrac{v_s + i_s \cdot R_a}{R_c + R_a}
\end{bmatrix}
$$