Electronics I for M.E.

Mid-Term Exam

SOLUTIONS

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1 Problem 1

1.1 Problem Statement

In the simple resistor-capacitor (RC) circuit shown in Figure 1 below, the input voltage is a sine wave,

$$\begin{aligned} v_{IN}(t) &= V_P \cdot \cos(\omega \cdot t) \\ &= \mathrm{Re}\{V_P \cdot \exp(j \cdot \omega \cdot t)\}. \\ &= \mathrm{Re}\{vz_{IN}(t)\}. \end{aligned}$$

Find the steady-state sine wave output voltage $v_{OUT}(t)$ as a function of the resistance R, the capacitance C, the peak input voltage V_P, the angular frequency ω, and time t. Use the concept of complex impedance.

Your solution should be of the form

$$\begin{aligned} v_{OUT}(t) &= \mathrm{Re}\{vz_{OUT}(t)\} \\ &= \mathrm{Re}\{G(\omega) \cdot vz_{IN}(t)\} \end{aligned}$$

where $G(\omega)$ is the ratio of simple complex polynomials in ω.

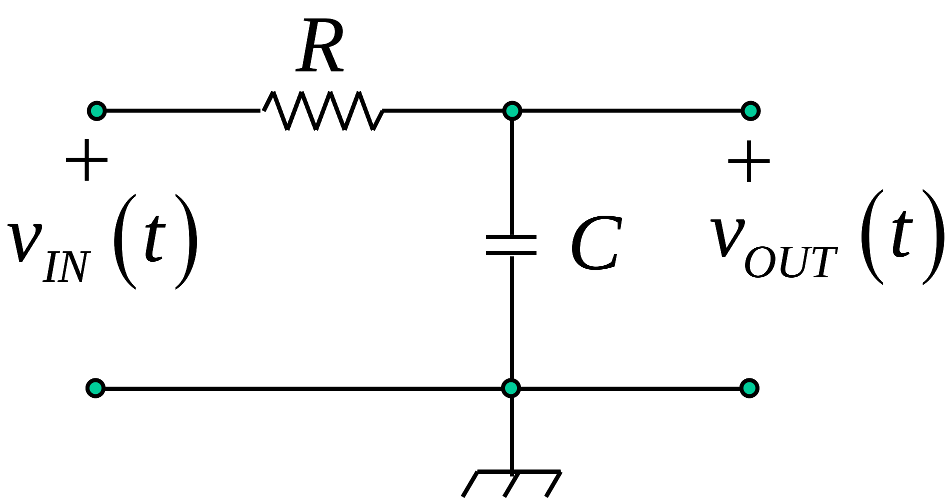

Figure 1. Simple RC Circuit for Problem 1.

1.2 Solution

At angular frequency ω, the capacitor has impedance

$$Z_C = \frac{1}{j \cdot \omega \cdot C}$$

The circuit of Figure 1 is a voltage divider. The output is

$$v_{OUT}(t) = \mathrm{Re}\left\{ \frac{Z_C}{R+Z_C} \cdot vz_{IN}(t) \right\} = \mathrm{Re}\left\{ \frac{1}{1+j\cdot\omega\cdot R\cdot C} \cdot vz_{IN}(t) \right\}.$$

We see that the transfer function is

$$G(\omega) = \frac{vz_{OUT}(t)}{vz_{IN}(t)} = \frac{1}{1+j\cdot\omega\cdot R\cdot C}.$$

2 Problem 2

2.1 Problem Statement

In Figure 2 below, we have a signal model of a bipolar junction transistor (BJT) in emitter follower configuration. We have the input in a Thévenin equivalent circuit with voltage $v_{IN}(t)$ and resistance R_B. Find the Thévenin equivalent circuit across the terminals x and y as algebraic expressions in terms of the values of the components and the current gain β.

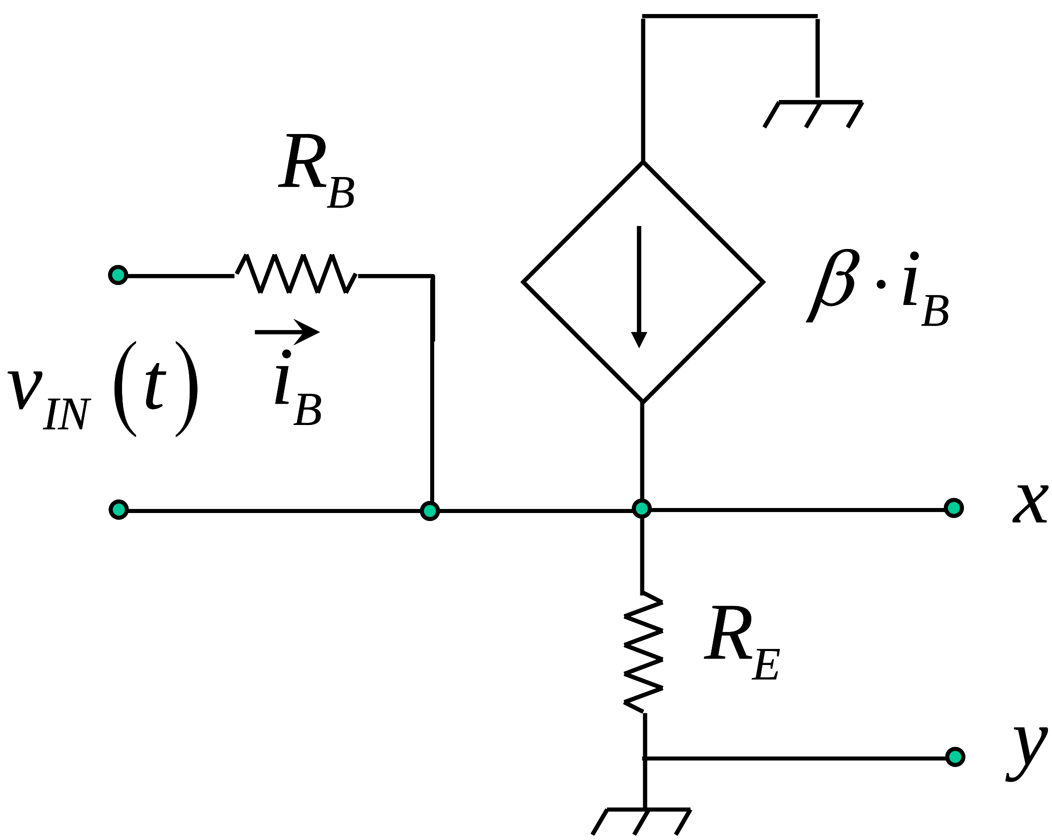

Figure 2. Signal Model of BJT in Emitter Follower Configuration for Problem 2.

2.2 Solution

The input voltage, as shown in the schematic, is relative to the emitter voltage, not ground. No one asked a question about this during the quiz and some worked the problem as if the input voltage $v_{IN}(t)$ was relative to ground like the lecture, homework and examples, but a few worked the problem as drawn in the schematic. Full credit is given for solving either configuration.

In addition, some students used the test source method. Thus there are two solutions to this problem that receive full credit, and two approaches to this solution were used. All four are given here.

2.2.1 First Configuration – Input Voltage Relative to Emitter Voltage

This is the configuration of the circuit as drawn in the quiz problem statement. Only a minority solved the problem this way. Most used an example from class that had a signal source relative to ground, which did not apply directly to this configuration.

2.2.1.1 Solving the Circuit

The solution is very simple if the input voltage $v_{IN}(t)$ is taken as relative to the emitter voltage $v_x(t)$ as drawn in Figure 2 because the base current i_B is a function only of the input voltage. We have

$$i_B = \frac{v_{IN}(t)}{R_B}$$

and, since i_B in the circuit as drawn does not flow thorough the emitter resistor R_E the only current that does flow through R_E is the current from the controlled source, and

$$v_x(t) = \beta \cdot i_B \cdot R_E = \frac{\beta \cdot R_E}{R_B} \cdot v_{IN}(t).$$

Since v_y is zero we have the open circuit voltage with $v_x(t)$. The short circuit current is simply the controlled source current,

$$i_{x,SC} = \beta \cdot i_B = \frac{\beta}{R_B} \cdot v_{IN}(t)$$

so we have the Thévenin equivalent resistance as

$$R_{xy,Th} = \frac{v_x(t)}{i_{x.SC}(t)} = R_E .$$

This simple result is due to the fact that there is no current path from terminal x to ground other than the emitter resistor. Thus when we drive the BJT with a voltage relative to the emitter, as we might do with a transformer secondary winding, we have a gain equation similar to that of an inverting amplifier, except for arithmetic sign, with the emitter resistor taking the place of the collector resistor.

2.2.1.2 The Test Source Method

The open circuit is as given in Section 2.2.1; the input voltage v_{IN} is taken as zero to allow application of the test source method to determine the impedance at the output terminals as before. The input terminals provide no path for current from the test source, and the current through the controlled source is zero, so the Thévenin equivalent resistance is simply

$$R_{Th} = R_E \,.$$

2.2.2 Second Configuration – Input Voltage Relative to Ground

The majority of the class applied examples from the lectures to solve this problem as if the input voltage was provided relative to ground, instead of relative to the emitter as the circuit was drawn in the problem statement. Correct solution for this configuration instead of the configuration given in Figure 2 is given full credit.

2.2.2.1 Solving the Circuit

The circuit is easily solved several ways. For example, the only voltage node is the voltage at point x. The node voltage equation is

$$v_x(t)\cdot\left(\frac{1}{R_B}+\frac{1}{R_E}\right)=\beta\cdot i_B+\frac{v_{IN}(t)}{R_B}=\beta\cdot\frac{v_{IN}(t)-v_x(t)}{R_B}+\frac{v_{IN}(t)}{R_B}$$
$$=\frac{1+\beta}{R_B}\cdot v_{IN}(t)-\frac{\beta}{R_B}\cdot v_x(t)$$

from which we can solve for $v_x(t)$ as

$$v_x(t)=\frac{\frac{1+\beta}{R_B}}{\frac{1+\beta}{R_B}+\frac{1}{R_E}}\cdot v_{IN}(t)=\frac{(1+\beta)\cdot R_E}{R_B+(1+\beta)\cdot R_E}\cdot v_{IN}(t)\,.$$

Since the terminal y is at ground potential, we have the open circuit voltage as $v_x(t)$.

The short circuit current is simple because with the terminal x shorted to ground, simplifying the base current equation. We find the base current $i_{B,SC}$ directly from Ohm's law as

$$i_{B,SC}(t)=\frac{v_{IN}(t)}{R_B}$$

and the short circuit current is the sum of this and $\beta\cdot i_{B,SC}$,

$$i_{X,SC}(t)=\frac{1+\beta}{R_B}\cdot v_{IN}(t)\,.$$

Thus the Thévenin resistance is

$$R_{xy,Th}=\frac{v_x(t)}{i_{x,SC}(t)}=\frac{1}{\frac{1+\beta}{R_B}+\frac{1}{R_E}}=\frac{R_B\cdot R_E}{R_B+(1+\beta)\cdot R_E}\,.$$

2.2.2.2 The Test Source Method

The test source method for finding the Thévenin equivalent provides the open circuit voltage by solving the circuit as given, and that solution is v_x as given above. The Thévenin resistance is found by taking v_{IN} as zero to provide a zero open circuit voltage, as is required to use the test source method, then solving a new circuit with a test source added at the output terminals.

With v_{IN} taken as zero, we have

$$i_B = -\frac{v_{Test}}{R_B}$$

and thus the current supplied by the test source into terminal x is

$$i_{Test} = \left(\frac{1+\beta}{R_B} + \frac{1}{R_E} \right) \cdot v_{Test}$$

so we have the Thévenin equivalent resistance as given above.

3 Problem 3

3.1 Problem Statement

A diode circuit is shown as Figure 3 to right. The diode has the v-i characteristic shown in Figure 4 below. Draw the load line on Figure 4 for $V_{IN} = 5\,V$ and $R = 100\,\Omega$. From the load line, find the voltage v_D across the diode and the current i_D through the diode.

3.2 Solution

The intersection of the load line and the diode v-i curve is

$$v_D = 0.68 \text{ Volts}$$
$$I_D = 0.043 \text{ A} = 43 \text{ mA} .$$

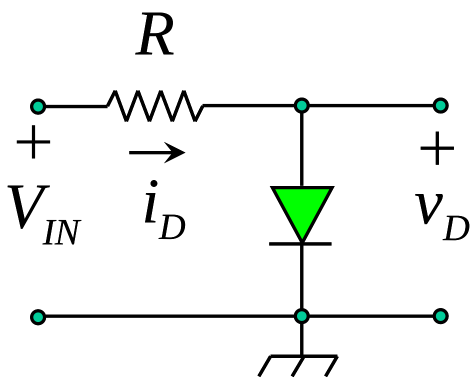

Figure 3. Circuit for Problem 3.

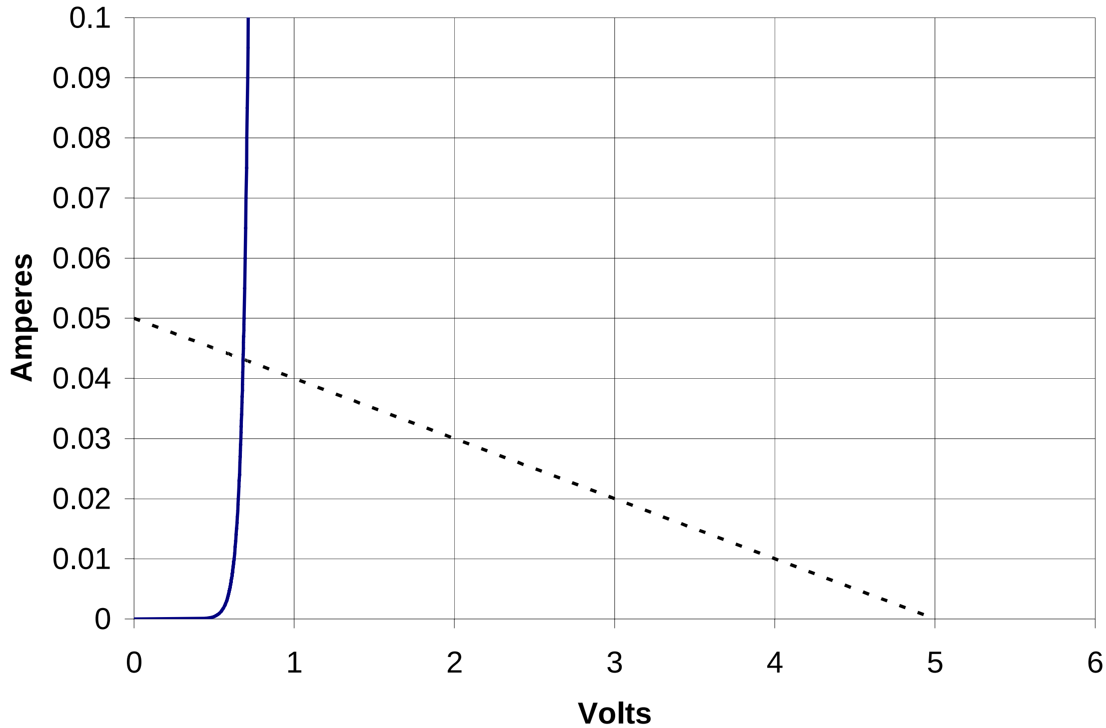

Figure 4. Diode v-i Curve [$v_T = 0.025\,V,\ \ I_S = 1\,nA,\ \ \eta = 1.55$]

4 Problem 4

4.1 Problem Statement

For a bipolar junction transistor (BJT) with v-i curves as shown in Figure 5, we have a collector load resistor with a resistance of $1\,k\Omega$, and a resistor in series with the base with resistance $100\,k\Omega$. The voltage drop from base to emitter is 0.7 Volts.

- Draw the load line.
- Find the output voltages V_{OUT} for input voltages V_{IN} of 0.7 Volts, 1.7 Volts, 2.7 Volts, 3.7 Volts, and 4.7 Volts.
- Draw a plot of output voltage (y axis) versus input voltage (x axis).

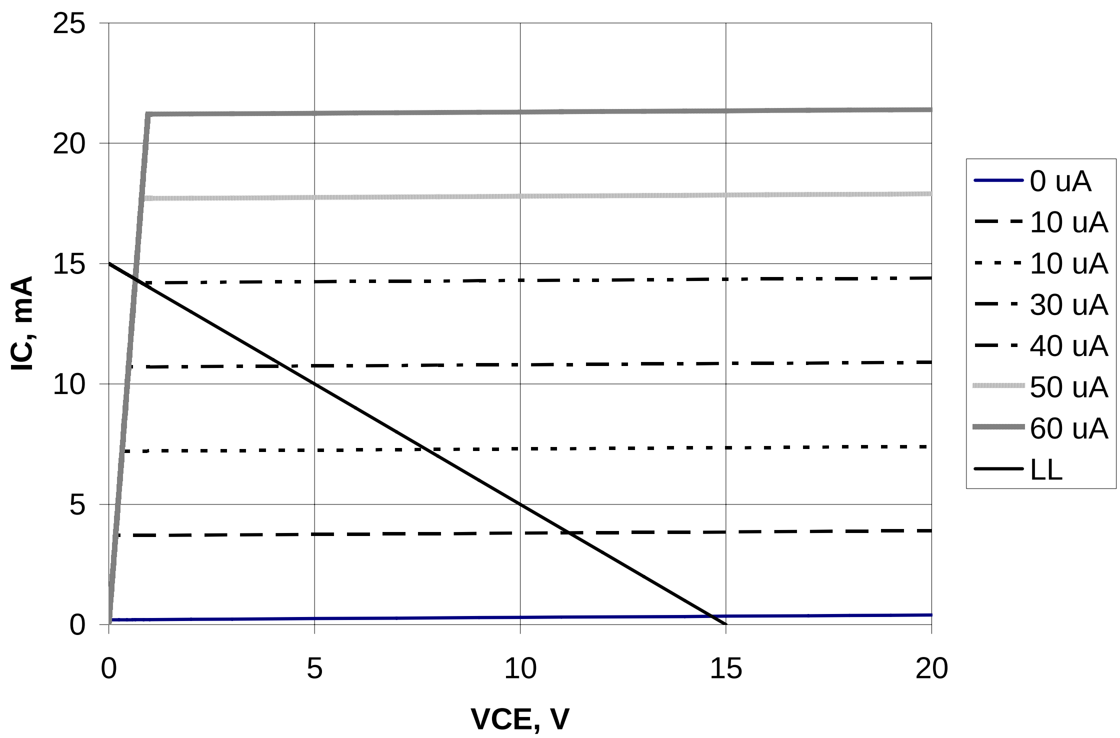

Figure 5. BJT v-i Curve for Problem 4.

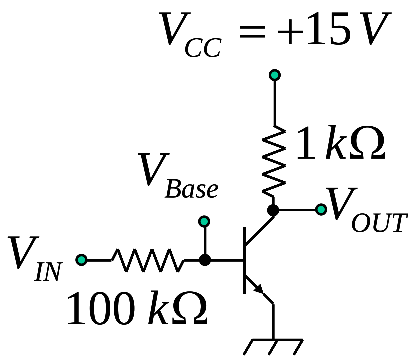

Figure 6. Simple BJT inverter circuit for Problem 4.

4.2 Solution

The load line is drawn on Figure 5. The base currents are found from the given values of V_{IN} from Ohm's law, noting that the voltage drop from base to emitter is 0;7 Volts, as stated in the problem. The table of values is

Table 1. Output vs. Input of Inverting Amplifier from Load Line.

V_{IN}	V_{IN} - 0.7 V	Coll Base Current, μA	Collector Voltage V_{OUT}
≤ 0.7 V	0.0 V	0	15 V
1.7 V	1.0 V	10	11.2 V
2.7 V	2.0 V	20	7.6 V
3.7 V	3.0 V	30	4.2 V
≥ 4.7 V	4.0 V	40	0.7 V

A plot of V_{OUT} versus V_{IN} is shown below as Figure 7.

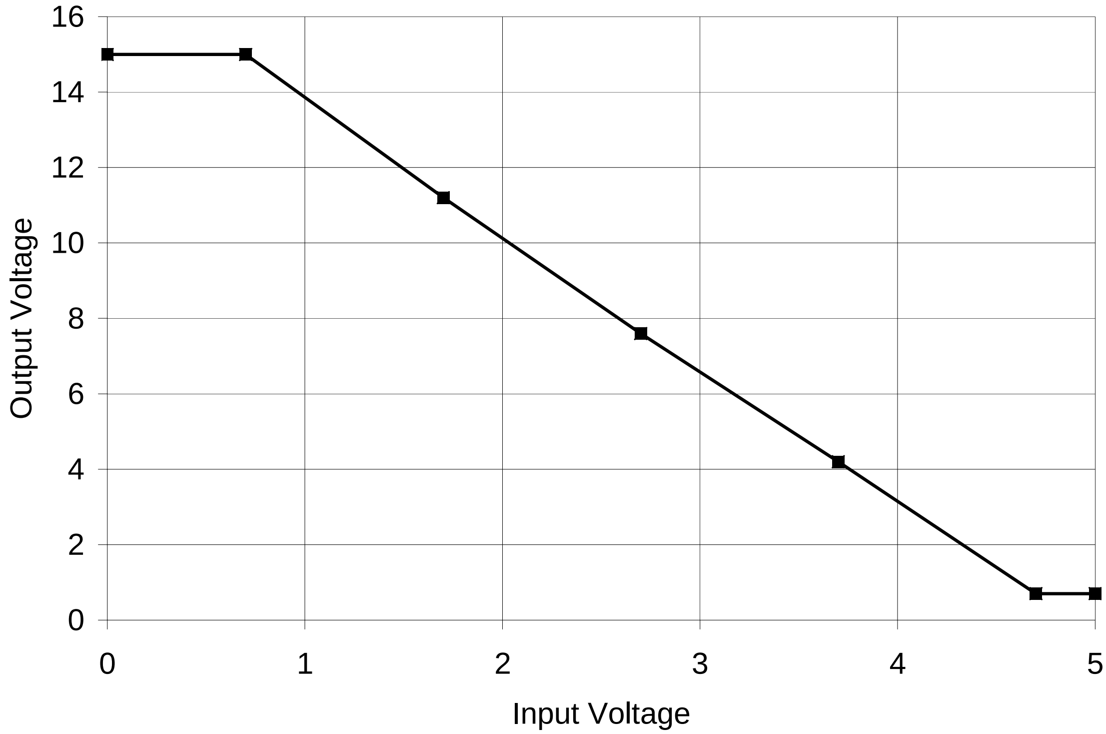

Figure 7. Output vs. Input for Inverting Amplifier.