Electronics I Quiz 1

Solutions

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1 Problem 1 (25%)

1.1 Problem Statement

In the circuit of Figure 1 below, use KVL and KCL to find a matrix equation that will solve for either the currents through each of the resistors, or the voltages at the marked Node 1 and Node 2, for the circuit below.

Do not use values for the components. Given an algebraic solution as a matrix equation of the form

(1.1) $$M \cdot \vec{x} = \vec{b}$$

where the elements of the matrix M are constants or functions of the passive component values, the elements of the vector $\vec{x}$ are the unknown voltages and currents, and the elements of the vector $\vec{b}$ are functions of the voltages and currents of the sources in the circuit and the values of the passive circuit elements. Your solution to this problem is the explicit algebraic equation for each of the elements of M, $\vec{x}$, and $\vec{b}$.

Give the matrix equation to solve for either the currents or the voltages, but not both. Do not invert the matrix to obtain the solutions.

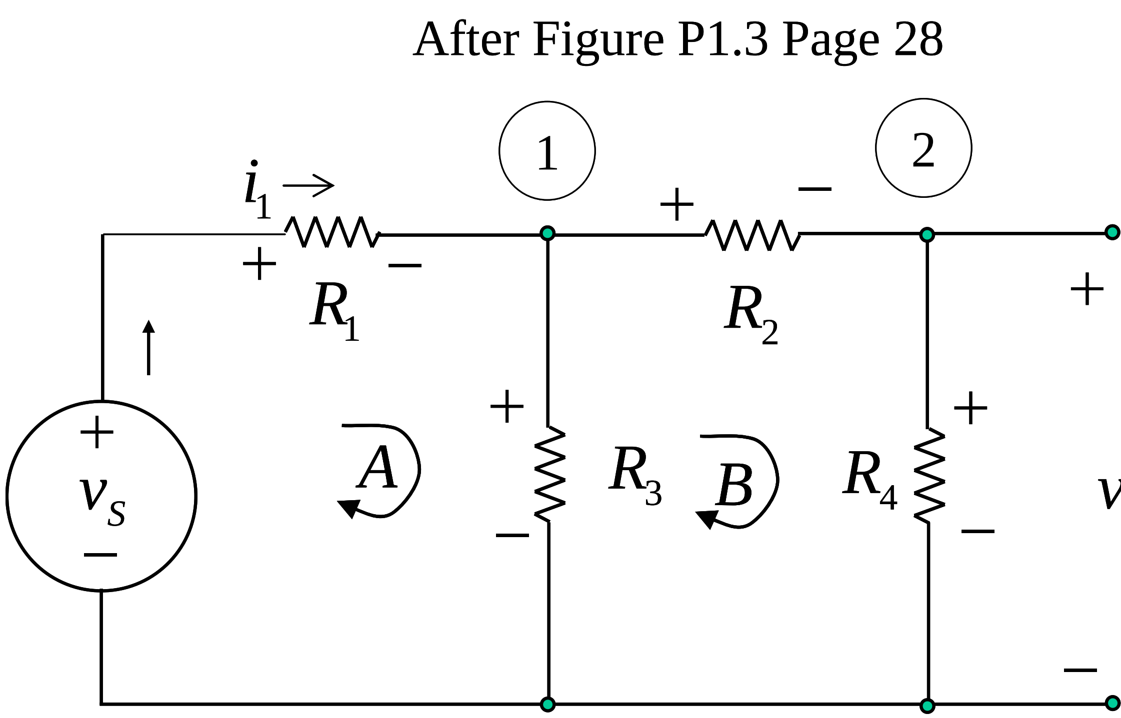

Figure 1. Circuit to Solve for Problem 1.

1.2 Solutions

In these solutions, we use the KCL by summing the currents out of node, and the voltages around each loop as the voltage drops across each element. The KVL convention is the voltage drop across each element in a loop.

1.2.1 Node Voltage Solution

The node voltage solution can be written by inspection using the techniques given in Lecture 1:

$$\begin{array}{c} \text{Node 1} \\ \text{Node 2} \end{array} \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_4} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{v_S}{R_1} \\ 0 \end{bmatrix} \tag{1.2}$$

A trick solution can be found by noting that there is only one real unknown voltage node, Node 1, and writing

$$\left(\frac{1}{R_1}+\frac{1}{R_2+R_4}+\frac{1}{R_3}\right)\cdot V_1 = \frac{v_S}{R_1} \tag{1.3}$$

This illustrates the power of the node voltage notation.

The determinant of the matrix is

$$Det = \frac{R_1\cdot\left(R_2+R_3+R_4\right)+R_3\cdot\left(R_2+R_4\right)}{R_1\cdot R_2\cdot R_3\cdot R_4} \tag{1.4}$$

The node voltages are

$$\begin{cases} V_1 = \dfrac{R_3\cdot\left(R_2+R_4\right)}{R_1\cdot\left(R_2+R_3+R_4\right)+R_3\cdot\left(R_2+R_4\right)}\cdot v_S \\ V_2 = \dfrac{R_3\cdot R_4}{R_1\cdot\left(R_2+R_3+R_4\right)+R_3\cdot\left(R_2+R_4\right)}\cdot v_S \end{cases} \tag{1.5}$$

The circuit is completely solved from the node voltages using voltage differences and Ohm's law. You can use the determinant and the node voltage solutions to check your method, if it differs from all of the three given here.

1.2.2 Resistor Currents or Mesh Solution

Note the signs marked on each of the resistors. We use Kirchhoff's current laws on the nodes.

$$\begin{cases} \text{Node 1} & -i_1+i_2+i_3=0 \\ \text{Node 2} & i_2-i_4=0 \end{cases} \tag{1.6}$$

Note that Node 2 simply provides us with the information that $i_2 = i_4$, which is the problem reduction that gives us the scalar equation for the node voltage notation method. Here we simply use it to find three currents instead of four.

To complete the three equations for the three unknown currents, we need Kirchhoff's voltage law, and we use Ohm's law to state it in terms of the unknown currents:

(1.7)
$$\begin{cases} \text{Loop A} & -v_S + i_1 \cdot R_1 + i_3 \cdot R_3 = 0 \\ \text{Loop B} & -i_3 \cdot R_3 + i_2 \cdot R_2 + i_2 \cdot R_4 = 0\,. \end{cases}$$

We collect these equations in matrix form as

(1.8)
$$\begin{array}{r} \\ \text{Node 1} \\ \text{Loop A} \\ \text{Loop B} \end{array} \begin{array}{c} \begin{array}{ccc} i_1 & i_2 & i_3 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 \\ R_1 & 0 & R_3 \\ 0 & R_2 + R_4 & -R_3 \end{bmatrix} \end{array} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ v_S \\ 0 \end{bmatrix}$$

1.2.3 Resistor Voltage Solution

We write the node equations using Ohm's law to write it in terms of the voltages across the resistors.

(1.9)
$$\begin{cases} \text{Node 1} & -\dfrac{v_1}{R_1} + \dfrac{v_2}{R_2} + \dfrac{v_3}{R_3} = 0 \\ \text{Node 2} & -\dfrac{v_2}{R_2} + \dfrac{v_4}{R_4} = 0 \end{cases}$$

Note that there is no clear advantage to eliminating a variable because the Node 2 equation is a little less than trivial in this notation. We have four unknown voltages and we need two more equations. We write the loop equations conventionally.

(1.10)
$$\begin{cases} \text{Loop A} & -v_S + v_1 + v_3 = 0 \\ \text{Loop B} & -v_3 + v_2 + v_4 = 0 \end{cases}$$

We now combine these equations in matrix format.

(1.11)
$$\begin{array}{r} \text{Node 1} \\ \text{Node 2} \\ \text{Loop A} \\ \text{Loop B} \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} -\dfrac{1}{R_1} & \dfrac{1}{R_2} & \dfrac{1}{R_3} & 0 \\ 0 & -\dfrac{1}{R_2} & 0 & \dfrac{1}{R_4} \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \end{array} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ v_S \\ 0 \end{bmatrix}$$

2 Problem 2 (25%)

2.1 Problem Statement

In the simple resistor-capacitor (RC) circuit shown in Figure 2 below, the input voltage is

(2.1)
$$\begin{aligned} v_{IN}(t) &= V_P \cdot \cos(\omega \cdot t) \\ &= \operatorname{Re}\left\{V_P \cdot \exp(j \cdot \omega \cdot t)\right\} \\ &= \operatorname{Re}\left\{vz_{IN}(t)\right\}. \end{aligned}$$

Find the steady-state output voltage $v_{OUT}(t)$ as a function of the resistance R, the capacitance C, the peak input voltage V_P, angular frequency ω, and time t. Use the concept of complex impedance.

Your solution should be of the form

(2.2)
$$\begin{aligned} v_{OUT}(t) &= \mathrm{Re}\{vz_{OUT}(t)\} \\ &= \mathrm{Re}\{G(\omega)\cdot vz_{IN}(t)\} \end{aligned}$$

where $G(\omega)$ is the ratio of simple complex polynomials in ω.

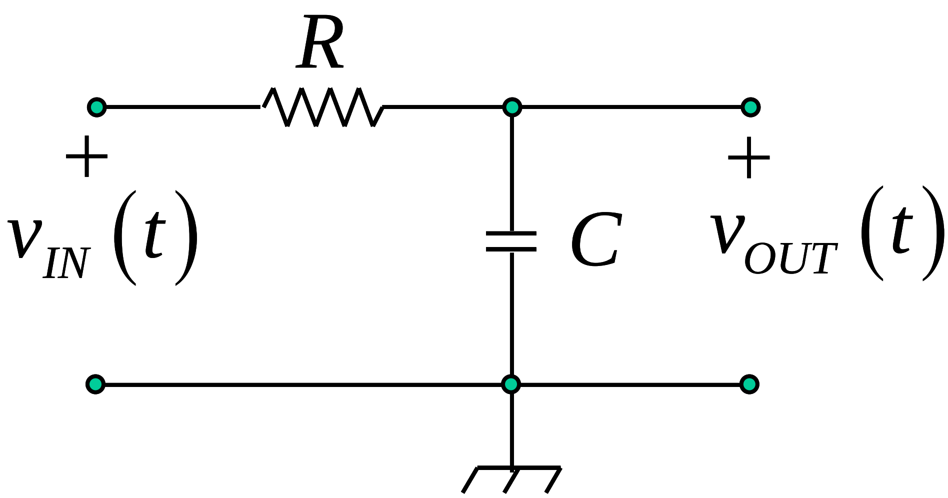

Figure 2. Simple RC Circuit for Problem 2.

2.2 Solution

The complex voltage notation is provided as part of the problem statement. The circuit, for a steady state sine wave, can be provided using the concept of capacitive impedance, and recognizing that the circuit is a simple voltage divider. The complex impedance of a capacitance C is

(2.3)
$$Z_C = \frac{1}{j\cdot\omega\cdot C}$$

so we have

(2.4)
$$\begin{aligned} vz_{OUT}(t) &= \frac{Z_C}{R+Z_C}\cdot vz_{IN}(t) = \frac{\dfrac{1}{j\cdot\omega\cdot C}}{R+\dfrac{1}{j\cdot\omega\cdot C}}\cdot vz(t) \\ &= \frac{1}{1+j\cdot\omega\cdot R\cdot C}\cdot vz(t) \end{aligned}$$

The output voltage is the real part of this equation,

(2.5)
$$v_{OUT}(t) = \mathrm{Re}\{vz_{OUT}(t)\} = \mathrm{Re}\left\{\frac{1}{1+j\cdot\omega\cdot R\cdot C}\cdot vz_{IN}(t)\right\}$$

and the transfer function $G(\omega)$ is

(2.6) $$G(\omega)=\frac{1}{1+j\cdot\omega\cdot R\cdot C}$$

3 Problem 3 (25%)

3.1 Problem Statement

A diode circuit is shown as Figure 3 below. The diode has the v-i characteristic shown in Figure 4 below. For $V_{IN}=6\,V$ and $R=120\,\Omega$, find the voltage v_D across the diode and the current i_D through the diode using a load line drawn on Figure 4. Then, compute the diode voltage and current using a constant voltage drop of $0.7\,V$ across the diode and compare the solutions.

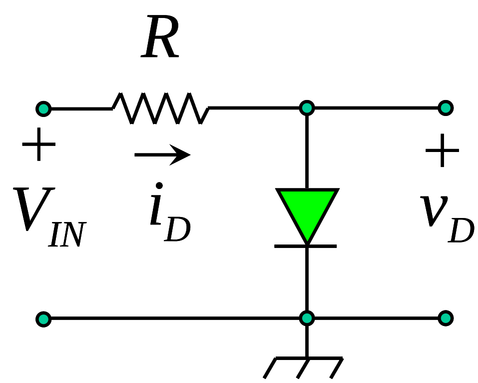

Figure 3. Circuit for Problem 3.

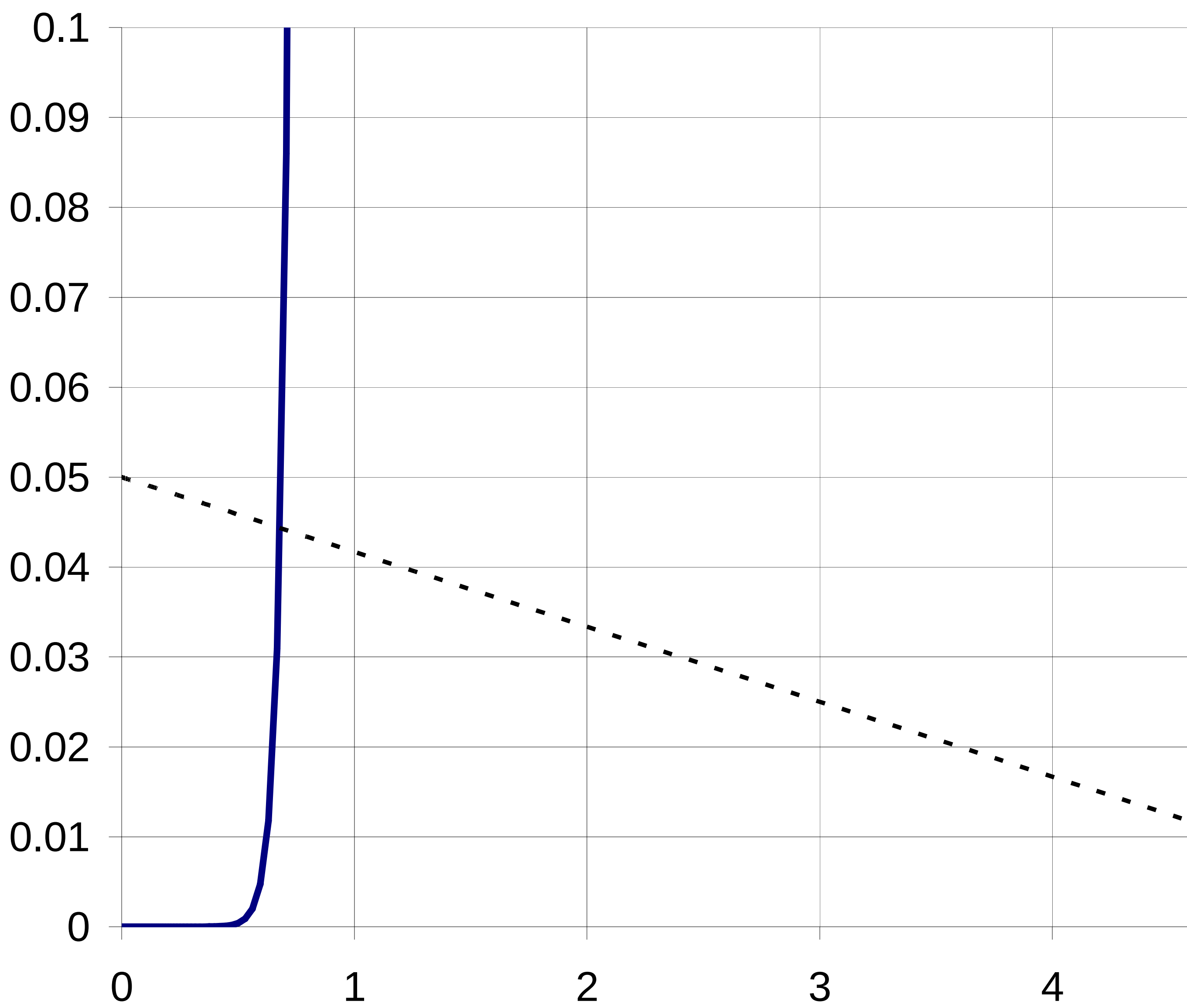

Figure 4. Diode v-i Curve [$v_T = 0.025\,V,\;\; I_S = 1\,nA,\;\; \eta = 1.55$ **]**

3.2 Solution

From the load line, the original Excel program using Goal Seek finds the intersection of the lines to be

(3.1) $$\begin{cases} v_D = 0.68\,V \\ i_D = 44.3\,mA. \end{cases}$$

The constant voltage drop model, for a diode voltage drop of $0.7\,V$, gives us a diode current of

$$i_D = \frac{6\,V - 0.7\,V}{120\,\Omega} = \frac{5.3\,V}{120\,\Omega} = 44.2\,mA. \tag{3.2}$$

Comparing the two solutions, we see that the differences are similar to those seen in different logarithmic diode models, depending on the parameters that are use for the saturation current I_S and emission coefficient η. For most practical purposes, the constant voltage model is sufficient.

4 Problem 4 (25%)

4.1 Problem Statement

In Figure 5 below, we have a signal model of a bipolar junction transistor (BJT) in emitter follower configuration. We have the input in a Thévenin equivalent circuit with voltage $v_{IN}(t)$ and resistance R_B. Find the Thévenin equivalent circuit across the terminals x and y as algebraic expressions in terms of the values of the components and the current gain β.

Using the equation that you have found, give the Thévenin voltage and resistance for

$$\begin{cases} v_{IN}(t) = V_P \cdot \cos(2\pi \cdot f \cdot t) \\ V_P = 1\,V \\ f = 1000\,Hz \\ R_B = 10\,k\Omega \\ R_E = 1.5\,k\Omega \\ \beta = 100 \end{cases} \tag{4.1}$$

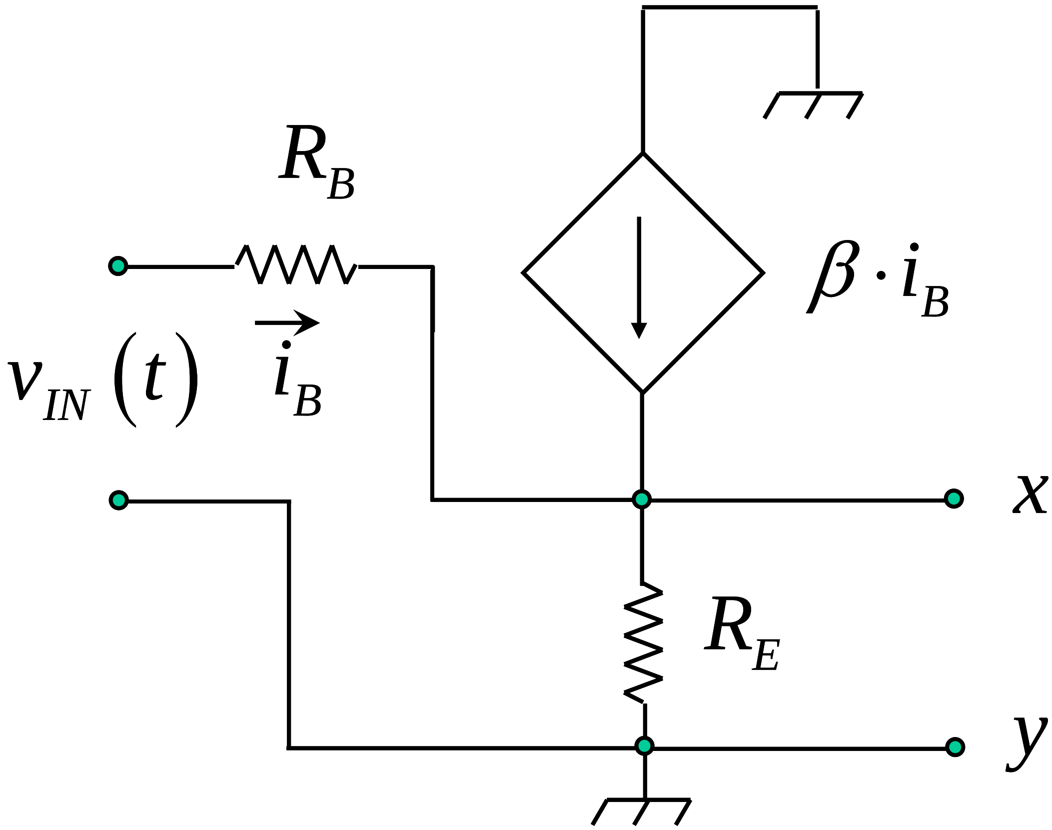

Figure 5. Signal Model of BJT in Emitter Follower Configuration

4.2 Direct Solution

We leverage what we learned from homework Problem 1.65 in this solution. The problem statement requires us to find the Thévenin equivalent as seen at the terminals x and y, which we will do by finding the open circuit voltage and short circuit current. We see that the current through R_E is $(1+\beta)$ times the current through R_B. We immediately have the short circuit current from Ohm's law because we have the voltage at the terminal x as zero for the short circuit condition, so, the currents out of node x give us

(4.2) $$i_{SC} = i_E = i_B + \beta \cdot i_B = \frac{v_{IN}(t)}{R_B} \cdot (1+\beta) = K \cdot \frac{v_{IN}(t)}{R_E}$$

where we define an internal circuit gain factor K as

(4.3) $$K = (1+\beta) \cdot \frac{R_E}{R_B}.$$

For the open circuit voltage, we find the current through R_B as

(4.4) $$i_B = \frac{v_{IN}(t) - v_x}{R_B}$$

so that the current through R_E is

(4.5)
$$\begin{aligned} i_E &= (1+\beta)\cdot i_B = (1+\beta)\cdot \frac{v_{IN}(t)-v_x}{R_B} \\ &= K\cdot \frac{v_{IN}(t)-v_x}{R_E} \end{aligned}$$

and we have the voltage across R_E, which is also the open circuit voltage, as

(4.6)
$$v_x = i_E \cdot R_E = K\cdot\left(v_{IN}(t)-v_x\right).$$

We solve this equation for v_x and obtain

(4.7)
$$v_x = \frac{K}{1+K}\cdot v_{IN}(t) = \frac{(1+\beta)\cdot R_E}{R_B+(1+\beta)\cdot R_E}\cdot v_{IN}(t).$$

We can now write the base current as

(4.8)
$$i_B = \frac{1}{1+K}\cdot v_{IN}(t) = \frac{R_B}{R_B+(1+\beta)\cdot R_E}\cdot v_{IN}(t).$$

The Thévenin resistance is

(4.9)
$$R_{TH} = \frac{v_x}{i_{SC}} = \frac{R_E}{1+K} = \frac{R_B\cdot R_E}{R_B+(1+\beta)\cdot R_E}$$

which is the parallel combination of R_E and $R_B/(1+\beta)$.

4.3 Test Source Solution

A number of people used the test source method, apparently because this method is featured in Example 1.5 pages 17 and 18, although that method is not required here because the open circuit voltage is not zero. We make the open circuit voltage zero by setting $v_{IN}(t)$ to zero and adding a test source. The circuit we analyze for the test source method is shown below as Figure 6.

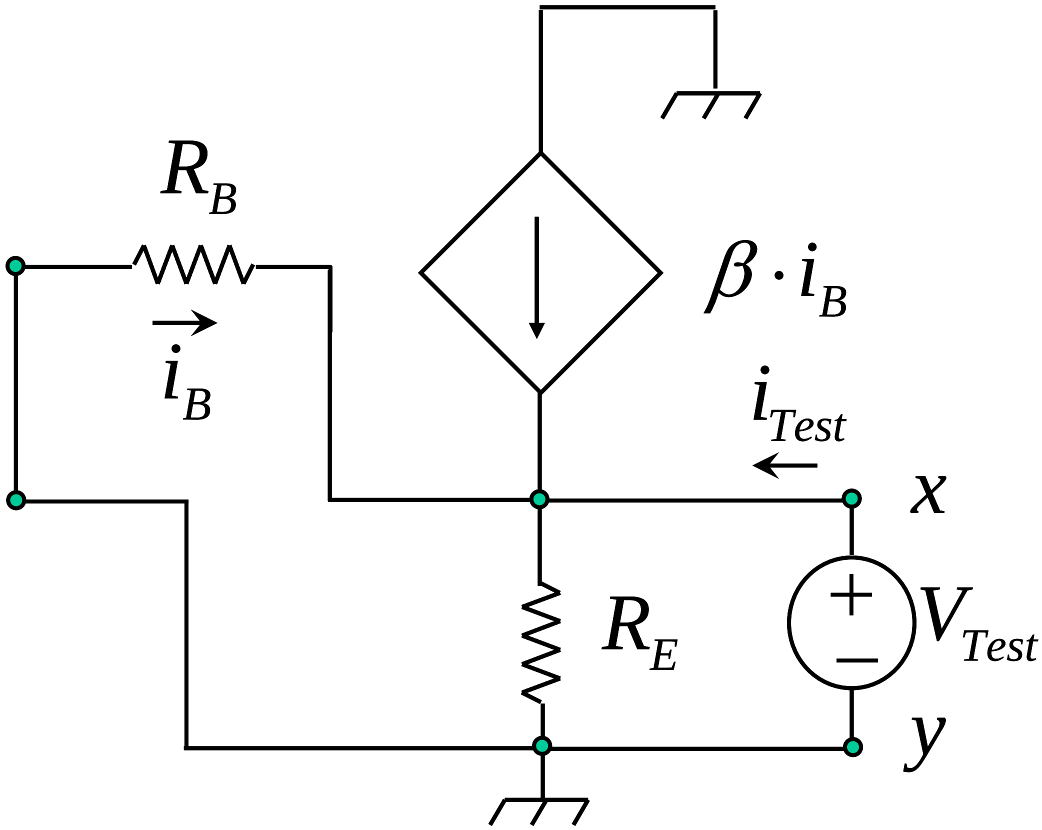

Figure 6. Circuit for Test Source Method.

We see that here

(4.10) $$i_B = -\frac{V_{Test}}{R_B}$$

and, summing the currents out of the only node in this circuit, node x,

(4.11) $$i_{Test} = -(1+\beta)\cdot i_B + \frac{V_{Test}}{R_E} = \left(\frac{1+\beta}{R_B} + \frac{1}{R_E}\right)\cdot V_{Test}$$

from which we have

(4.12) $$R_{TH} = \frac{V_{Test}}{i_{Test}} = \frac{1}{\frac{1+\beta}{R_B} + \frac{1}{R_E}} = \frac{R_B \cdot R_E}{R_B + (1+\beta)\cdot R_E}$$

which is in agreement with (4.9). For this solution to be complete, the open circuit voltage as given by (4.7) must also be provided.

4.4 Numerical Solution

For the numerical parameters given in the problem statement, we have

(4.13) $$K = (1+\beta)\cdot\frac{R_E}{R_B} = 101\cdot\frac{1.5}{10} = 15.15$$

so that the Thévenin voltage is

(4.14) $$v_x = \frac{15.15}{16.15} \cdot v_{IN}(t) = 0.93808 \cdot v_{IN}(t)$$

and the Thévenin resistance is

(4.15) $$R_{TH} = \frac{R_E}{1+K} = \frac{1500\,\Omega}{16.15} = 92.8793\,\Omega\,.$$

Thus we see that the emitter follower provides a voltage gain very nearly unity with a very low output impedance.